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NCAR/TN-xxx+STR

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## Preface

The present document describes the update to an analytical method of solving for variables near the land-atmosphere interface that will appear in the Community Land Model version 4 (CLM4). This document is an addendum to the standard CLM technical description (Oleson *et al.* 2004). This work was supported in part by the … program through grant ….

Samuel Levis

Boulder, 20 April 2006

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## 1. Introduction

The Community Land Model (CLM) solves a set of simultaneous equations once per model time step *n*. The unknowns in this set of equations include near-surface prognostic temperature and humidity variables for time step $n + 1$.

The near-surface state responds to the conditions prescribed by an atmospheric data set when the CLM operates in offline mode or simulated by an atmospheric general circulation model (GCM) when the CLM operates in coupled mode. In the latter case, CLM's calculated sensible and latent heat fluxes are passed to the GCM to establish two-way land-atmosphere interactions.

CLM versions prior to version 4 employ an iterative scheme of solving for near-surface temperature and humidity (Oleson *et al.* 2004). Here we document the model update to an analytical method of solving for the same variables using a matrix solver (Vidale & Stöckli 2005). This update will appear in CLM version 4. The analytical solution of this matrix of equations will simplify large sections of the code and will allow for an easier implementation of water isotope tracers in CLM.

The CLM already solves for soil and snow temperature and moisture using a matrix solver. This will remain with the slight change that the temperature of the top layer of soil (or snow if present) will now be solved by the new, prognostic canopy air space matrix. As a result the existing matrix solving for soil and snow temperature will now solve for one less layer.

## 2. The Equations and their Physical Basis

This section includes subsections numbered by matrix row, where each row corresponds to an equation. All rows together form the simultaneous set of equations solving for a number of unknowns equal to the number of equations. The complete matrix appears in section 3. Symbols for all variables are consistent with Oleson *et al.* (2004).

Each equation is presented in three forms: (a) the physical form, (b) a series of forms following algebraic transformations, and (c) the matrix coefficient form. The algebraic transformations assume an "explicit coefficient/implicit temperature" numerical scheme (Kalnay and Kanamitsu 1988). "Explicit coefficient" means that we use the resistance terms ($r_{ah}$, $r_{aw}$, $r_b$) calculated at time step $n$, while "implicit temperature" means that the variables on the right hand side (RHS) of the equations are from time step $n + 1$.

## 2.1 Solving for $T_s$

Eq. 1 solves for $T_s$ (K), the canopy air space temperature. Eq. 1 states that the change in $T_s$ per time increment $\Delta t$ (s) between time steps $n$ and $n + 1$ is directly proportional to the sum of sensible heat fluxes (W m$^{-2}$) from the ground, $H_g$, the vegetation, $H_v$, and the GCM's reference height ($z_{atm,h} \approx 30$ m above the ground), $H$, to the height of the canopy air space ($z_{0h} + d$ (Oleson *et al.* 2004)) (Vidale & Stöckli 2005). The first two sensible heat fluxes are positive into and the third is positive away from the canopy air space:

$$C_s \frac{\Delta T_s}{\Delta t} = H_g^{n+1} + H_v^{n+1} - H^{n+1} \qquad \text{(Eq. 1a)}$$

where $C_s$ is the canopy air space heat capacity equal to $\rho_{atm} C_p \Delta z$, $\rho_{atm}$ is the density of atmospheric, or moist, air (kg m$^{-3}$), and $C_p$ is the specific heat capacity of dry air (J kg$^{-1}$

K[-1]). Δz is the greater of 4 m and the difference between the top and bottom heights of the canopy. If Δz tended to zero, $C_s$ would tend to zero and the prognostic form of Eq. 1a would reduce to the diagnostic expression used in CLM prior to version 4 (Vidale & Stöckli 2005). Eq. 1a′ is shown as a reminder of an assumption that ceases to be true in CLM version 4:

$$\lim_{C_s \to 0} C_s \frac{\Delta T_s}{\Delta t} = 0 \Rightarrow H^{n+1} = H_g^{n+1} + H_v^{n+1} \qquad \text{(Eq. 1a}'\text{)}$$

Do not confuse $C_s$ with the variable in Oleson *et al.* (2004) that represents the turbulent transfer coefficient between soil and canopy air.

Starting from Eq. 1a, we carry the $n + 1$ sensible heat flux terms to the LHS, add the corresponding $n$ terms to both sides of the equation, expand all terms, and rearrange the LHS by variable instead of by time step:

$$C_s \frac{\Delta T_s}{\Delta t} + H^{n+1} - H_g^{n+1} - H_v^{n+1} = 0$$

$$C_s \frac{\Delta T_s}{\Delta t} + H^{n+1} - H^n - H_g^{n+1} + H_g^n - H_v^{n+1} + H_v^n = H_v^n + H_g^n - H^n$$

$$\begin{aligned}
& C_s \frac{\Delta T_s}{\Delta t} - \frac{\rho_{atm} C_p}{r_{ah}}\left(\bar{\theta}_{atm}^{n+1} - T_s^{n+1}\right) + \frac{\rho_{atm} C_p}{r_{ah}}\left(\bar{\theta}_{atm}^{n} - T_s^{n}\right) \\
& + \frac{\rho_{atm} C_p}{r_{ah'}}\left(T_s^{n+1} - T_g^{n+1}\right) - \frac{\rho_{atm} C_p}{r_{ah'}}\left(T_s^{n} - T_g^{n}\right) \\
& + \rho_{atm} C_p \frac{L+S}{r_b}\left(T_s^{n+1} - T_v^{n+1}\right) - \rho_{atm} C_p \frac{L+S}{r_b}\left(T_s^{n} - T_v^{n}\right) \\
& = -\rho_{atm} C_p \frac{L+S}{r_b}\left(T_s^{n} - T_v^{n}\right) - \frac{\rho_{atm} C_p}{r_{ah'}}\left(T_s^{n} - T_g^{n}\right) + \frac{\rho_{atm} C_p}{r_{ah}}\left(\bar{\theta}_{atm}^{n} - T_s^{n}\right)
\end{aligned} \qquad \text{(Eq. 1b)}$$

$$\left(\frac{C_s}{\Delta t}+\frac{\rho_{atm}C_p}{r_{ah}}+\rho_{atm}C_p\frac{L+S}{r_b}+\frac{\rho_{atm}C_p}{r_{ah'}}\right)\left(T_s^{n+1}-T_s^n\right)-\frac{\rho_{atm}C_p}{r_{ah}}\left(\overline{\theta}_{atm}^{n+1}-\overline{\theta}_{atm}^{n}\right)$$
$$-\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_g^{n+1}-T_g^n\right)-\rho_{atm}C_p\frac{L+S}{r_b}\left(T_v^{n+1}-T_v^n\right)$$
$$=-\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^n-T_v^n\right)-\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^n-T_g^n\right)+\frac{\rho_{atm}C_p}{r_{ah}}\left(\overline{\theta}_{atm}^{n}-T_s^n\right)$$

where $T_g$ and $T_v$ are the ground and leaf temperatures (K), $\overline{\theta}_{atm}$ is the potential temperature (K) at the GCM's reference height, $L$ and $S$ are the exposed leaf and stem area index values ($m^2$ leaf or stem surface $m^{-2}$ ground), $r_{ah}$ is the aerodynamic resistance to sensible heat transfer (s $m^{-1}$) between CLM's canopy air space and the GCM's reference height, $r_b$ is the leaf boundary layer resistance (s $m^{-1}$), and $r_{ah}'$ is the aerodynamic resistance to heat transfer (s $m^{-1}$) between the ground and the canopy air space. Whether in offline or coupled mode, CLM assumes that a dataset will provide or an AGCM will calculate $\overline{\theta}_{atm}^{n+1}$. Therefore, CLM does not calculate $\overline{\theta}_{atm}^{n+1}$ and assumes instead that $\overline{\theta}_{atm}^{n+1}-\overline{\theta}_{atm}^{n}=0$ to solve the matrix. The corresponding term in Eq. 1b drops out.

$T_s$, $T_g$, and $\overline{\theta}_{atm}$, are column level, while $T_v$, $C_s$, and the resistance terms are plant functional type (pft) level variables. Generalizing Eq. 1b to include multiple pfts per column and substituting $C_s$ with $\rho_{atm}C_p\Delta z$ gives:

$$
\begin{aligned}
&\sum_{j=1}^{npft}\left[(wt)_j\left(\frac{\Delta z_j}{\Delta t}+\frac{1}{(r_{ah})_j}+\frac{L_j+S_j}{(r_b)_j}+\frac{1}{(r_{ah'})_j}\right)\rho_{atm}C_p\right]\left(T_s^{n+1}-T_s^n\right)\\
&-\sum_{j=1}^{npft}\left[(wt)_j\frac{\rho_{atm}C_p}{(r_{ah'})_j}\right]\left(T_g^{n+1}-T_g^n\right)\\
&-\sum_{j=1}^{npft}\left[(wt)_j\frac{L_j+S_j}{(r_b)_j}\rho_{atm}C_p\left[(T_v^{n+1})_j-(T_v^n)_j\right]\right]\\
&=\sum_{j=1}^{npft}\left[(wt)_j\left(\begin{array}{l}-\rho_{atm}C_p\dfrac{L_j+S_j}{(r_b)_j}\left(T_s^n-(T_v^n)_j\right)\\-\dfrac{\rho_{atm}C_p}{(r_{ah'})_j}\left(T_s^n-T_g^n\right)+\dfrac{\rho_{atm}C_p}{(r_{ah})_j}\left(\overline{\theta}_{atm}^n-T_s^n\right)\end{array}\right)\right]
\end{aligned}
\qquad \text{(Eq. 1b}'\text{)}
$$

where $j$ is the pft index ranging from 1 to *npft* (the number of pfts present in the column) and $(wt)_j$ is the fraction of the column occupied by pft $j$, where $\sum_{j=1}^{npft}(wt)_j=1$. CLM includes bare ground in the same column as the vegetation and gives it a pft index. The fraction of the column with bare ground has $L_j=0$ and $S_j=0$.

In matrix coefficient form, Eq. 1b′ becomes:

$$
\begin{aligned}
C_{T_s}^1 &=\sum_{j=1}^{npft}\left[(wt)_j\left(\frac{\Delta z_j}{2\Delta t}+\frac{1}{(r_{ah})_j}+\frac{L_j+S_j}{(r_b)_j}+\frac{1}{(r_{ah'})_j}\right)\rho_{atm}C_p\right]\\
C_{(T_v)_j}^1 &=-(wt)_j\frac{L_j+S_j}{(r_b)_j}\rho_{atm}C_p \quad \text{for} \quad j=1,2,\ldots,npft\\
C_{T_g}^1 &=-\sum_{j=1}^{npft}\left[(wt)_j\frac{\rho_{atm}C_p}{(r_{ah'})_j}\right]\\
F_{T_s} &=\sum_{j=1}^{npft}\left[(wt)_j\left(\begin{array}{l}-\rho_{atm}C_p\dfrac{L_j+S_j}{(r_b)_j}\left(T_s^n-(T_v^n)_j\right)\\-\dfrac{\rho_{atm}C_p}{(r_{ah'})_j}\left(T_s^n-T_g^n\right)+\dfrac{\rho_{atm}C_p}{(r_{ah})_j}\left(\overline{\theta}_{atm}^n-T_s^n\right)\end{array}\right)\right]
\end{aligned}
\qquad \text{(Eq. 1c)}
$$

where $C^1_{T_s}$ is the matrix coefficient in row 1 that is multiplied by $\Delta T_s$, $C^1_{(T_v)_j}$ is multiplied by $\Delta(T_v)_j$ (for $j$ = 1, 2, …, *npft*), and $C^1_{T_g}$ is multiplied by $\Delta T_g$. A smoothing filter is introduced by multiplying the time step, $\Delta t$, by a factor of 2. $F_{T_s}$ is the RHS term of Eq. 1.

**2.2 Solving for $q_s$**

Eq. 2 solves for $q_s$, the specific humidity (kg water vapor $kg^{-1}$ air) of the canopy air space. Eq. 2 states that the change in $q_s$ with respect to time is directly proportional to the sum of latent heat fluxes (W $m^{-2}$) from the ground, $\lambda E_g$, the vegetation, $\lambda E_v$, and the GCM's reference height ($z_{atm,w} = z_{atm,h}$ (Oleson *et al.* 2004)), $\lambda E$, to the canopy air space height ($z_{0w} + d = z_{0h} + d$ (Oleson *et al.* 2004)) (Vidale & Stöckli 2005). The first two latent heat fluxes are positive into and the third is positive away from the canopy air space:

$$\rho_{atm} \lambda \Delta z \frac{\Delta q_s}{\Delta t} = \lambda E_g^{n+1} + \lambda E_v^{n+1} - \lambda E^{n+1} \qquad \text{(Eq. 2a)}$$

where $\lambda E_v = \lambda E_v^t + \lambda E_v^w$, i.e. the sum of transpiration and canopy evaporation, and $\lambda$ (J $kg^{-1}$) is the latent heat of sublimation if the water content of the top soil/snow layer is all ice and no liquid; $\lambda$ is the latent heat of vaporization when liquid water is present in the soil or when calculating plant ***evapo?***transpiration. Without the factor $\lambda$, the units of Eq. 2a would have been water vapor flux units (kg $m^{-2}$ $s^{-1}$) instead of energy flux units (W $m^{-2}$). Other terms in Eq. 2a have been defined previously.

If Δz tended to zero, the prognostic form of Eq. 2a would reduce to the diagnostic expression used in CLM prior to version 4 (not shown but, e.g., see Eq. 1a′ in section 2.1).

Next we carry the $n + 1$ latent heat flux terms to the LHS, add the corresponding $n$ terms to both sides of the equation, expand all terms, and rearrange by variable instead of by time step:

$$\rho_{atm}\lambda\Delta z\frac{\Delta q_s}{\Delta t}+\lambda E^{n+1}-\lambda E_g^{n+1}-\lambda E_v^{n+1}=0$$

$$\rho_{atm}\lambda\Delta z\frac{\Delta q_s}{\Delta t}+\lambda E^{n+1}-\lambda E^{n}-\lambda E_g^{n+1}+\lambda E_g^{n}-\lambda E_v^{n+1}+\lambda E_v^{n}=\lambda E_v^{n}+\lambda E_g^{n}-\lambda E^{n}$$

$$\begin{aligned}
&\rho_{atm}\lambda\Delta z\frac{\Delta q_s}{\Delta t}-\frac{\rho_{atm}\lambda}{r_{aw}}\left(q_{atm}^{n+1}-q_s^{n+1}\right)+\frac{\rho_{atm}\lambda}{r_{aw}}\left(q_{atm}^{n}-q_s^{n}\right)+\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^{n+1}-q_g^{n+1}\right)-\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^{n}-q_g^{n}\right)\\
&+\rho_{atm}\lambda\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(q_s^{n+1}-q_{sat}^{T_v^{n+1}}\right)\\
&-\rho_{atm}\lambda\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(q_s^{n}-q_{sat}^{T_v^{n}}\right)\\
&=-\rho_{atm}\lambda\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(q_s^{n}-q_{sat}^{T_v^{n}}\right)\\
&-\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^{n}-q_g^{n}\right)+\frac{\rho_{atm}\lambda}{r_{aw}}\left(q_{atm}^{n}-q_s^{n}\right)
\end{aligned}$$

$$\begin{aligned}
&\rho_{atm}\lambda\left\{\frac{\Delta z}{\Delta t}+\frac{1}{r_{aw}}+f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)+\frac{1}{r_{aw'}}\right\}\left(q_s^{n+1}-q_s^{n}\right)\\
&-\rho_{atm}\lambda\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(q_{sat}^{T_v^{n+1}}-q_{sat}^{T_v^{n}}\right)\\
&-\frac{\rho_{atm}\lambda}{r_{aw}}\left(q_{atm}^{n+1}-q_{atm}^{n}\right)-\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_g^{n+1}-q_g^{n}\right)\\
&=-\rho_{atm}\lambda\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(q_s^{n}-q_{sat}^{T_v^{n}}\right)\\
&-\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^{n}-q_g^{n}\right)+\frac{\rho_{atm}\lambda}{r_{aw}}\left(q_{atm}^{n}-q_s^{n}\right)
\end{aligned}\qquad\text{(Eq. 2b)}$$

where $f_{wet}$ is the wetted fraction of the canopy (leaves and stems) and $f_{dry}$ is the fraction of leaves that are dry and able to photosynthesize ($f_{wet}$ and $f_{dry}$ are defined mathematically in Oleson *et al.* (2004) and $f_{dry} \neq 1 - f_{wet}$ in general). When the soil moisture function that limits transpiration $\beta_t \leq 1 \times 10^{-10}$, $f_{dry} = 0$. When dew is present, $f_{dry} = 0$ and $f_{wet} = 1$. $L^{sun}$ and $L^{sha}$ are the sunlit and shaded components of $L$ ($m^2$ $m^{-2}$), $r_s^{sun}$ and $r_s^{sha}$ are the sunlit and shaded stomatal resistances (s $m^{-1}$), $r_{aw}$ is the aerodynamic resistance to water vapor transfer (s $m^{-1}$) between the canopy air space and the GCM's reference height, $r_{aw}'$ is the aerodynamic resistance to water vapor transfer (s $m^{-1}$) between the ground and the canopy air space, $q_g$ is the specific humidity (kg $kg^{-1}$) at the ground, and $q_{sat}^{T_v}$ is the saturated specific humidity (kg $kg^{-1}$) at temperature $T_v$. Whether in offline or coupled mode, CLM assumes that a dataset will provide or an AGCM will calculate $q_{atm}^{n+1}$. Therefore, CLM does not calculate $q_{atm}^{n+1}$ and assumes instead that $q_{atm}^{n+1} - q_{atm}^{n} = 0$ to solve the matrix, so the corresponding term in Eq. 2b drops out.

Assuming that $\dfrac{dq_{sat}^{T}}{dT} = \dfrac{q_{sat}^{T^{n+1}} - q_{sat}^{T^{n}}}{T^{n+1} - T^{n}}$, where $q_{sat}^{T}$ is the saturated specific humidity at temperature $T$, and assuming that $\dfrac{dq_g}{dT_g} = \alpha \dfrac{dq_{sat}^{T_g}}{dT_g}$ given that $q_g = \alpha q_{sat}^{T_g}$, where $q_g$ is the specific humidity at the ground as a function of the saturated specific humidity at the ground (section 5.2 of Oleson et al. (2004)), we substitute the terms $q_g^{n+1} - q_g^{n}$ and $q_{sat}^{T_v^{n+1}} - q_{sat}^{T_v^{n}}$ to get Eq. 2b′:

$$
\begin{aligned}
&\rho_{atm}\lambda\left\{\frac{\Delta z}{\Delta t}+\frac{1}{r_{aw}}+f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)+\frac{1}{r_{aw'}}\right\}\left(q_s^{n+1}-q_s^n\right)\\
&-\rho_{atm}\lambda\frac{dq_{sat}^{T_v}}{dT_v}\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(T_v^{n+1}-T_v^n\right)\\
&-\frac{\rho_{atm}\lambda}{r_{aw'}}\frac{dq_g}{dT_g}\left(T_g^{n+1}-T_g^n\right) \qquad \text{(Eq. 2b}'\text{)}\\
&=-\rho_{atm}\lambda\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(q_s^n-q_{sat}^{T_v^n}\right)\\
&-\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^n-q_g^n\right)+\frac{\rho_{atm}\lambda}{r_{aw}}\left(q_{atm}^n-q_s^n\right)
\end{aligned}
$$

$T_g$, $q_s$, and $q_{atm}$, are column level, while $T_v$ and the resistance terms are pft level variables. Generalizing Eq. 2b′ to include multiple pfts per column gives:

$$
\begin{aligned}
&\sum_{j=1}^{npft}\left[(wt)_j\rho_{atm}\lambda\left\{\begin{matrix}\frac{\Delta z_j}{\Delta t}+\frac{1}{(r_{aw})_j}+\frac{1}{(r_{aw'})_j}+(f_{wet})_j\frac{L_j+S_j}{(r_b)_j}\\ +\frac{(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j}+\frac{L_j^{sha}}{(r_b)_j+(r_s^{sha})_j}\right)\end{matrix}\right\}\right]\left(q_s^{n+1}-q_s^n\right)\\
&-\sum_{j=1}^{npft}\left[(wt)_j\rho_{atm}\lambda\frac{dq_{sat}^{(T_v)_j}}{d(T_v)_j}\left\{\begin{matrix}(f_{wet})_j\frac{L_j+S_j}{(r_b)_j}\\ +\frac{(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j}+\frac{L_j^{sha}}{(r_b)_j+(r_s^{sha})_j}\right)\end{matrix}\right\}\left[(T_v^{n+1})_j-(T_v^n)_j\right]\right]\\
&-\sum_{j=1}^{npft}\left[(wt)_j\frac{\rho_{atm}\lambda}{(r_{aw'})_j}\frac{dq_g}{dT_g}\right]\left(T_g^{n+1}-T_g^n\right) \qquad \text{(Eq.2b")}\\
&=\sum_{j=1}^{npft}\left[(wt)_j\left(\begin{matrix}-\rho_{atm}\lambda\left\{(f_{wet})_j\frac{L_j+S_j}{(r_b)_j}+\frac{(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j}+\frac{L_j^{sha}}{(r_b)_j+(r_s^{sha})_j}\right)\right\}\left(q_s^n-q_{sat}^{(T_v^n)_j}\right)\\ -\frac{\rho_{atm}\lambda}{(r_{aw'})_j}\left(q_s^n-q_g^n\right)+\frac{\rho_{atm}\lambda}{(r_{aw})_j}\left(q_{atm}^n-q_s^n\right)\end{matrix}\right)\right]
\end{aligned}
$$

In matrix coefficient form, Eq. 2b″ becomes:

$$C_{q_s}^2 = \sum_{j=1}^{npft} \left[ (wt)_j \rho_{atm} \lambda \begin{Bmatrix} \frac{\Delta z_j}{2\Delta t} + \frac{1}{(r_{aw})_j} + \frac{1}{(r_{aw'})_j} + (f_{wet})_j \frac{L_j + S_j}{(r_b)_j} \\ + \frac{(f_{dry})_j}{L_j} \left( \frac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j + (r_s^{sha})_j} \right) \end{Bmatrix} \right]$$

$$C_{(T_v)_j}^2 = -(wt)_j \rho_{atm} \lambda \frac{dq_{sat}^{(T_v)_j}}{d(T_v)_j} \begin{Bmatrix} (f_{wet})_j \frac{L_j + S_j}{(r_b)_j} \\ + \frac{(f_{dry})_j}{L_j} \left( \frac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b) + (r_s^{sha})_j} \right) \end{Bmatrix} \text{ for } j = 1,2,\ldots,npft$$

$$C_{T_g}^2 = -\sum_{j=1}^{npft} \left[ (wt)_j \frac{\rho_{atm}\lambda}{(r_{aw'})_j} \frac{dq_g}{dT_g} \right] \qquad \text{(Eq.2c)}$$

$$F_{q_s} = \sum_{j=1}^{npft} \left[ (wt)_j \begin{pmatrix} -\rho_{atm}\lambda \left\{ (f_{wet})_j \frac{L_j + S_j}{(r_b)_j} + \frac{(f_{dry})_j}{L_j} \left( \frac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j + (r_s^{sha})_j} \right) \right\} \left( q_s^n - q_{sat}^{(T_v^n)_j} \right) \\ - \frac{\rho_{atm}\lambda}{(r_{aw'})_j} \left( q_s^n - q_g^n \right) + \frac{\rho_{atm}\lambda}{(r_{aw})_j} \left( q_{atm}^n - q_s^n \right) \end{pmatrix} \right]$$

where $C_{q_s}^2$ is the matrix coefficient in row 2 that is multiplied by $\Delta q_s$, $C_{(T_v)_j}^2$ is multiplied by $\Delta(T_v)_j$, and $C_{T_g}^2$ is multiplied by $\Delta T_g$. A smoothing filter is introduced by multiplying the time step, $\Delta t$, by a factor of 2. $F_{q_s}$ is the RHS term of Eq. 2. Although $\lambda$ could cancel out of all the terms in Eq. 2 as latent heat of vaporization, $\lambda$ could also represent the latent heat of sublimation in $C_{T_g}^2$ if the top soil/snow layer's moisture is all ice. Therefore, we keep $\lambda$ in all the terms of Eq. 2.

## 2.3 Solving for $T_v$

Eq. 3 states that vegetation temperature, $T_v$ (K), changes with time as a function of the net energy available to the vegetation (W m$^{-2}$), accounting for radiation and heat flux terms, as well as for changes in the vegetation's net longwave radiation. The radiation terms include the vegetation-absorbed net solar, $\vec{S}_v$ (positive into vegetation), and net longwave radiation, $\vec{L}_v$ (positive away from vegetation). The heat flux terms include the sensible and latent heat fluxes, $H_v$ and $\lambda E_v$ (positive away from vegetation). The change in the vegetation's net longwave radiation from time step *n* to *n* + 1 with respect to temperature is given by $\left.\frac{d\vec{L}_v}{dT_g}\right|_n \Delta T_g \delta_{veg} + \left.\frac{d\vec{L}_v}{dT_v}\right|_n \Delta T_v \delta_{veg}$ (positive away from vegetation).

$$C_v \frac{\Delta T_v}{\Delta t} = \vec{S}_v^n - \vec{L}_v^n - H_v^{n+1} - \lambda E_v^{n+1} - \left.\frac{d\vec{L}_v}{dT_g}\right|_n \Delta T_g \delta_{veg} - \left.\frac{d\vec{L}_v}{dT_v}\right|_n \Delta T_v \delta_{veg} \qquad \text{(Eq. 3a)}$$

where $C_v$ (J m$^{-2}$ K$^{-1}$) is the heat capacity of the vegetation equal to $(L+S)C_{liq}W_{l+s} + C_{liq}W_{can}$, where $C_{liq}$ is the specific heat capacity of water (J kg$^{-1}$ K$^{-1}$), $W_{l+s}$ is the amount of water in leaves and stems set to 0.2 kg m$^{-2}$ leaf and stem area, $W_{can}$ is the amount of water on the canopy per unit area of ground (kg m$^{-2}$), and $\delta_{veg}$ is a step function equal to zero for $L + S < 0.05$ and equal to one otherwise.

As in previous sections, now we transform Eq. 3a to Eq. 3b. At this time we also replace $\left.\frac{d\vec{L}_v}{dT_g}\right|_n$ and $\left.\frac{d\vec{L}_v}{dT_v}\right|_n$ with $-4\varepsilon_v\varepsilon_g\sigma\left(T_g^n\right)^3$ and $4\left[2-\varepsilon_v\left(1-\varepsilon_g\right)\right]\varepsilon_v\sigma\left(T_v^n\right)^3$, respectively:

$$C_v \frac{\Delta T_v}{\Delta t} + H_v^{n+1} - H_v^n + \lambda E_v^{n+1} - \lambda E_v^n + \left.\frac{d\vec{L}_v}{dT_g}\right|_n \Delta T_g \delta_{veg} + \left.\frac{d\vec{L}_v}{dT_v}\right|_n \Delta T_v \delta_{veg} = \vec{S}_v^n - \vec{L}_v^n - H_v^n - \lambda E_v^n$$

$$
\begin{aligned}
&\frac{C_v}{\Delta t}\left(T_v^{n+1}-T_v^n\right)-\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^{n+1}-T_v^{n+1}\right)+\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^n-T_v^n\right)\\
&-\rho_{atm}\lambda\left[f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_s^{n+1}-q_{sat}^{T_v^{n+1}}\right)\\
&+\rho_{atm}\lambda\left[f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_s^{n}-q_{sat}^{T_v^{n}}\right)\\
&+4\left[2-\varepsilon_v\left(1-\varepsilon_g\right)\right]\varepsilon_v\sigma\left(T_v^n\right)^3\delta_{veg}\left(T_v^{n+1}-T_v^n\right)-4\varepsilon_v\varepsilon_g\sigma\left(T_g^n\right)^3\delta_{veg}\left(T_g^{n+1}-T_g^n\right)\\
&=\vec{S}_v^n-\vec{L}_v^n+\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^n-T_v^n\right)+\rho_{atm}\lambda\left[f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_s^{n}-q_{sat}^{T_v^{n}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&\left[\frac{C_v}{\Delta t}+4\left[2-\varepsilon_v\left(1-\varepsilon_g\right)\right]\varepsilon_v\sigma\left(T_v^n\right)^3\delta_{veg}+\rho_{atm}C_p\frac{L+S}{r_b}\right]\left(T_v^{n+1}-T_v^n\right)-\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^{n+1}-T_s^n\right)\\
&+\rho_{atm}\lambda\left[f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_{sat}^{T_v^{n+1}}-q_{sat}^{T_v^{n}}\right)\\
&-\rho_{atm}\lambda\left[f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_s^{n+1}-q_s^{n}\right)\\
&-4\varepsilon_v\varepsilon_g\sigma\left(T_g^n\right)^3\delta_{veg}\left(T_g^{n+1}-T_g^n\right)\\
&=\vec{S}_v^n-\vec{L}_v^n+\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^n-T_v^n\right)+\rho_{atm}\lambda\left[f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_s^{n}-q_{sat}^{T_v^{n}}\right)
\end{aligned}
\qquad \text{(Eq. 3b)}
$$

where $\varepsilon_v$ and $\varepsilon_g$ are the vegetation and ground emissivities, and $\sigma$ is the Stefan-Boltzmann constant (W m$^{-2}$ K$^{-4}$).

As done in Eq. 2b, we next substitute $q_{sat}^{T_v^{n+1}}-q_{sat}^{T_v^{n}}$ with $\frac{dq_{sat}^{T_v}}{dT_v}\left(T_v^{n+1}-T_v^n\right)$:

$$
\begin{aligned}
&\left[\frac{C_v}{\Delta t} + 4\left[2-\varepsilon_v\left(1-\varepsilon_g\right)\right]\varepsilon_v\sigma\left(T_v^n\right)^3\delta_{veg} + \rho_{atm}C_p\frac{L+S}{r_b}\right.\\
&\left.+\rho_{atm}\lambda\frac{dq_{sat}^{T_v}}{dT_v}\left\{f_{wet}\frac{L+S}{r_b} + \frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}} + \frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\right]\left(T_v^{n+1}-T_v^n\right)\\
&-\rho_{atm}\lambda\left[f_{wet}\frac{L+S}{r_b} + \frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}} + \frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_s^{n+1}-q_s^n\right) \qquad \text{(Eq. 3b')}\\
&-\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^{n+1}-T_s^n\right) - 4\varepsilon_v\varepsilon_g\sigma\left(T_g^n\right)^3\delta_{veg}\left(T_g^{n+1}-T_g^n\right)\\
&=\vec{S}_v^n - \vec{L}_v^n + \rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^n - T_v^n\right) + \rho_{atm}\lambda\left[f_{wet}\frac{L+S}{r_b} + \frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}} + \frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_s^n - q_{sat}^{T_v^n}\right)
\end{aligned}
$$

$T_s$, $q_s$, and $T_g$ are column level, while $T_v$ and the resistance terms are pft level variables. Generalizing Eq. 3b′ to include multiple pfts per column gives:

$$
\begin{aligned}
&\left[\frac{(C_v)_j}{\Delta t} + 4\left[2-(\varepsilon_v)_j\left(1-\varepsilon_g\right)\right](\varepsilon_v)_j\sigma\left(T_v^n\right)_j^3(\delta_{veg})_j + \rho_{atm}C_p\frac{L_j+S_j}{(r_b)_j}\right.\\
&\left.+\rho_{atm}\lambda\frac{dq_{sat}^{(T_v)_j}}{d(T_v)_j}\left\{(f_{wet})_j\frac{L_j+S_j}{(r_b)_j} + \frac{(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j+(r_s^{sha})_j}\right)\right\}\right]\left[(T_v^{n+1})_j-(T_v^n)_j\right]\\
&-\rho_{atm}\lambda\left[(f_{wet})_j\frac{L_j+S_j}{(r_b)_j} + \frac{(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j+(r_s^{sha})_j}\right)\right]\left(q_s^{n+1}-q_s^n\right)\\
&-\rho_{atm}C_p\frac{L_j+S_j}{(r_b)_j}\left(T_s^{n+1}-T_s^n\right) - 4(\varepsilon_v)_j\varepsilon_g\sigma\left(T_g^n\right)^3(\delta_{veg})_j\left(T_g^{n+1}-T_g^n\right) \qquad \text{(Eq. 3b'')}\\
&=(\vec{S}_v^n)_j - (\vec{L}_v^n)_j + \rho_{atm}C_p\frac{L_j+S_j}{(r_b)_j}\left(T_s^n-(T_v^n)_j\right)\\
&+\rho_{atm}\lambda\left[(f_{wet})_j\frac{L_j+S_j}{(r_b)_j} + \frac{(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j+(r_s^{sha})_j}\right)\right]\left(q_s^n - q_{sat}^{(T_v^n)_j}\right)
\end{aligned}
$$

for $j = 1, 2, \ldots, npft$. There are as many equations solving for $T_v$ as pfts in the column. For bare ground, all terms in this equation reduce to zero, so the equation can be omitted from the matrix.

In matrix coefficient form, Eq. 3b″ becomes:

$$C_{T_s}^{2+j} = -\rho_{atm} C_p \frac{L_j + S_j}{(r_b)_j}$$

$$C_{q_s}^{2+j} = -\rho_{atm} \lambda \left\{ (f_{wet})_j \frac{L_j + S_j}{(r_b)_j} + \frac{(f_{dry})_j}{L_j} \left( \frac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j + (r_s^{sha})_j} \right) \right\}$$

$$C_{(T_v)_j}^{2+j} = \frac{(C_v)_j}{2\Delta t} + 4\left[2 - (\varepsilon_v)_j \left(1 - \varepsilon_g\right)\right](\varepsilon_v)_j \sigma \left(T_v^n\right)_j^3 (\delta_{veg})_j + \rho_{atm} C_p \frac{L_j + S_j}{(r_b)_j} \qquad \text{(Eq. 3c)}$$

$$+ \rho_{atm} \lambda \frac{dq_{sat}^{(T_v)_j}}{d(T_v)_j} \left\{ (f_{wet})_j \frac{L_j + S_j}{(r_b)_j} + \frac{(f_{dry})_j}{L_j} \left( \frac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j + (r_s^{sha})_j} \right) \right\}$$

$$C_{T_g}^{2+j} = -4(\varepsilon_v)_j \varepsilon_g \sigma \left(T_g^n\right)^3 (\delta_{veg})_j$$

$$F_{(T_v)_j}\Big|_{j=1}^{npft} = (\vec{S}_v^n)_j - (\vec{L}_v^n)_j + \rho_{atm} C_p \frac{L_j + S_j}{(r_b)_j} \left(T_s^n - (T_v^n)_j\right)$$

$$+ \rho_{atm} \lambda \left[ (f_{wet})_j \frac{L_j + S_j}{(r_b)_j} + \frac{(f_{dry})_j}{L_j} \left( \frac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j + (r_s^{sha})_j} \right) \right] \left(q_s^n - q_{sat}^{(T_v^n)_j}\right)$$

for *j* = 1, 2, …, *npft*. $C_{T_s}^{2+j}$ is the matrix coefficient in row 2 + *j* that is multiplied by $\Delta T_s$, $C_{q_s}^{2+j}$ is multiplied by $\Delta q_s$, $C_{(T_v)_j}^{2+j}$ is multiplied by $\Delta (T_v)_j$, and $C_{T_g}^{2+j}$ is multiplied by $\Delta T_g$. A smoothing filter is introduced by multiplying the time step, $\Delta t$, by a factor of 2. $F_{(T_v)_j}$ is the RHS term of Eq. 3.

The matrix solves for $(T_v)_j$ for all the pfts present in a column. For simplicity we continue section 2 as though *npft* equals one. Therefore, this section (2.3) discussed matrix row 3, the following section (2.4) will discuss matrix row 4, and so on.

## 2.4 Solving for $T_g$

In CLM, properties of the ground correspond to properties of the top soil layer (or snow layer if snow is present). Eq. 4 solves for $T_g$, the ground temperature (K), stating that $\frac{\Delta T_g}{\Delta t}$ is a function of the net energy available to the top soil/snow layer (W m$^{-2}$). As in Eq. 3, we include in net energy the radiation and heat flux terms, as well as changes in the ground's net longwave radiation. The radiation terms include ground-absorbed net solar, $\vec{S}_g$ (positive into the top soil/snow layer), and net longwave radiation, $\vec{L}_g$ (positive away from top soil/snow layer). The heat flux terms include sensible, latent, and soil heat fluxes, $H_g$, $\lambda E_g$, and $F_{1+snl}$ (positive away from top soil/snow layer). The change in the ground's net longwave radiation from time step $n$ to $n + 1$ with respect to temperature is given by $\left.\frac{d\vec{L}_g}{dT_g}\right|_n \Delta T_g + \left.\frac{d\vec{L}_g}{dT_v}\right|_n \Delta T_v$ (positive away from top soil/snow layer):

$$c_{1+snl}\Delta z_{i*}\frac{\Delta T_g}{\Delta t} = \vec{S}_g^n - \vec{L}_g^n - H_g^{n+1} - \lambda E_g^{n+1} - F_{1+snl}^{n+1} - \left.\frac{d\vec{L}_g}{dT_g}\right|_n \Delta T_g - \left.\frac{d\vec{L}_g}{dT_v}\right|_n \Delta T_v \qquad \text{(Eq. 4a)}$$

where $c_{1+snl}$ (J m$^{-3}$ K$^{-1}$) is the volumetric heat capacity of the top soil/snow layer (index 1+*snl*) and *snl* is the number of snow layers ranging from 0 to –5. With no snow the index for the top soil layer is 1, while with five layers of snow the index for the top snow layer is –4. $\Delta z_{i*}$ (m) is the top soil/snow layer thickness (Eq. 6.29 in Oleson *et al.* (2004)) indexed differently to indicate a numerical adjustment specific to the top layer. This adjustment intends to lower the heat capacity of the top layer to justify clm's assumption that $T_g$ and $T_1$, the temperature of the top layer are one and the same.

Transformations similar to the ones used in sections 2.2 and 2.3 lead from Eq. 4a to Eq. 4b. Here $\left.\frac{d\vec{L}_g}{dT_g}\right|_n$ and $\left.\frac{d\vec{L}_g}{dT_v}\right|_n$ are replaced with $4\varepsilon_g\sigma\left(T_g^n\right)^3$ and $-4\varepsilon_v\varepsilon_g\delta_{veg}\sigma\left(T_v^n\right)^3$, respectively. Also $q_g^{n+1} - q_g^n$ is replaced with $\frac{dq_g}{dT_g}\left(T_g^{n+1} - T_g^n\right)$, assuming that $\frac{dq_g}{dT_g} = \alpha \frac{dq_{sat}^{T_g}}{dT_g}$ as in Eq. 2b′. Note that $T_{2+snl}$ is known for time step $n$ only because the soil/snow temperature matrix is solved separately from the prognostic canopy air space matrix:

$$
\begin{aligned}
& c_{1+snl}\Delta z_{i*}\frac{\Delta T_g}{\Delta t} + \left.\frac{d\vec{L}_g}{dT_g}\right|_n \Delta T_g + \left.\frac{d\vec{L}_g}{dT_v}\right|_n \Delta T_v \\
& + H_g^{n+1} - H_g^n + \lambda E_g^{n+1} - \lambda E_g^n + F_{1+snl}^{n+1} - F_{1+snl}^n = \vec{S}_g^n - \vec{L}_g^n - H_g^n - \lambda E_g^n - F_{1+snl}^n
\end{aligned}
$$

$$
\begin{aligned}
& c_{1+snl}\Delta z_{i*}\frac{\Delta T_g}{\Delta t} + 4\varepsilon_g\sigma\left(T_g^n\right)^3\left(T_g^{n+1} - T_g^n\right) - 4\varepsilon_v\varepsilon_g\sigma\left(T_v^n\right)^3\delta_{veg}\left(T_v^{n+1} - T_v^n\right) \\
& - \frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^{n+1} - T_g^{n+1}\right) + \frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^n - T_g^n\right) - \frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^{n+1} - q_g^{n+1}\right) + \frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^n - q_g^n\right) \\
& - \frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl} - z_{1+snl}}\left(T_g^{n+1} - T_{2+snl}^n\right) + \frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl} - z_{1+snl}}\left(T_g^n - T_{2+snl}^n\right) \\
& = \vec{S}_g^n - \vec{L}_g^n + \frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^n - T_g^n\right) + \frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^n - q_g^n\right) + \frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl} - z_{1+snl}}\left(T_g^n - T_{2+snl}^n\right)
\end{aligned}
$$

$$
\begin{aligned}
& c_{1+snl}\Delta z_{i*}\frac{\Delta T_g}{\Delta t} + \left[\frac{\rho_{atm}C_p}{r_{ah'}} + 4\varepsilon_g\sigma\left(T_g^n\right)^3 - \frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl} - z_{1+snl}}\right]\left(T_g^{n+1} - T_g^n\right) \\
& - \frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^{n+1} - T_s^n\right) - \frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^{n+1} - q_s^n\right) + \frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_g^{n+1} - q_g^n\right) \\
& - 4\varepsilon_v\varepsilon_g\sigma\left(T_v^n\right)^3\delta_{veg}\left(T_v^{n+1} - T_v^n\right) \\
& = \vec{S}_g^n - \vec{L}_g^n + \frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^n - T_g^n\right) + \frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^n - q_g^n\right) + \frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl} - z_{1+snl}}\left(T_g^n - T_{2+snl}^n\right)
\end{aligned}
\qquad \text{(Eq. 4b)}
$$

$$
\left[\frac{c_{1+snl}\Delta z_{i*}}{\Delta t}+\frac{\rho_{atm}C_p}{r_{ah'}}+\frac{\rho_{atm}\lambda}{r_{aw'}}\frac{dq_g}{dT_g}+4\varepsilon_g\sigma\left(T_g^n\right)^3-\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\right]\left(T_g^{n+1}-T_g^n\right)
$$

$$
-4\varepsilon_v\varepsilon_g\sigma\left(T_v^n\right)^3\delta_{veg}\left(T_v^{n+1}-T_v^n\right)-\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^{n+1}-T_s^n\right)-\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^{n+1}-q_s^n\right)
$$

$$
=\vec{S}_g^n-\vec{L}_g^n+\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^n-T_g^n\right)+\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^n-q_g^n\right)+\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\left(T_g^n-T_{2+snl}^n\right)
$$

where the coefficient $\lambda$ was defined in section 2.2, while $\lambda[z_{h,1+snl}]$ (W m$^{-1}$ K$^{-1}$) is the thermal conductivity at the interface between the top and second soil/snow layers, $z_{h,1+snl}$ (m) refers to the depth of that interface, while $z_{1+snl}$ and $z_{2+snl}$ (m) are the depths of the top and second from the top soil/snow layers, respectively.

$T_s$, $q_s$, and $T_g$ are column level, while $T_v$ and the resistance terms are pft level variables. Generalizing Eq. 4b for multiple pfts per column gives:

$$
\sum_{j=1}^{npft}\left[(wt)_j\left\{\frac{c_{1+snl}\Delta z_{i*}}{\Delta t}+\frac{\rho_{atm}C_p}{(r_{ah'})_j}+\frac{\rho_{atm}\lambda}{(r_{aw'})_j}\frac{dq_g}{dT_g}+4\varepsilon_g\sigma\left(T_g^n\right)^3-\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\right\}\right]\left(T_g^{n+1}-T_g^n\right)
$$

$$
-\sum_{j=1}^{npft}\left[(wt)_j4\varepsilon_v\varepsilon_g\sigma\left(T_v^n\right)_j^3(\delta_{veg})_j\left[(T_v^{n+1})_j-(T_v^n)_j\right]\right]
$$

$$
-\sum_{j=1}^{npft}\left[(wt)_j\frac{\rho_{atm}C_p}{(r_{ah'})_j}\right]\left(T_s^{n+1}-T_s^n\right)-\sum_{j=1}^{npft}\left[(wt)_j\frac{\rho_{atm}\lambda}{(r_{aw'})_j}\right]\left(q_s^{n+1}-q_s^n\right) \qquad \text{(Eq. 4b′)}
$$

$$
=\sum_{j=1}^{npft}\left[(wt)_j\left(\begin{array}{l}\vec{S}_g^n-\vec{L}_g^n+\frac{\rho_{atm}C_p}{(r_{ah'})_j}\left(T_s^n-T_g^n\right)+\frac{\rho_{atm}\lambda}{(r_{aw'})_j}\left(q_s^n-q_g^n\right)\\+\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\left(T_g^n-T_{2+snl}^n\right)\end{array}\right)\right]
$$

In matrix coefficient form, Eq. 4b′ becomes:

$$C_{T_s}^4 = -\sum_{j=1}^{npft}\left[(wt)_j \frac{\rho_{atm} C_p}{(r_{ah'})_j}\right]$$

$$C_{q_s}^4 = -\sum_{j=1}^{npft}\left[(wt)_j \frac{\rho_{atm} \lambda}{(r_{aw'})_j}\right]$$

$$C_{(T_v)_j}^4 \Big|_{j=1}^{npft} = -(wt)_j 4\varepsilon_v \varepsilon_g \sigma \left(T_v^n\right)_j^3 (\delta_{veg})_j \quad \text{for} \quad j = 1,2,\ldots,npft$$

$$C_{T_g}^4 = \sum_{j=1}^{npft}\left[(wt)_j \left\{\frac{c_{1+snl}\Delta z_{i*}}{2\Delta t} + \frac{\rho_{atm} C_p}{(r_{ah'})_j} + \frac{\rho_{atm}\lambda}{(r_{aw'})_j}\frac{dq_g}{dT_g} + 4\varepsilon_g \sigma \left(T_g^n\right)^3 - \frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl} - z_{1+snl}}\right\}\right]$$

$$F_{T_g} = \sum_{j=1}^{npft}\left[(wt)_j \left(\begin{array}{l} \vec{S}_g^n - \vec{L}_g^n + \frac{\rho_{atm} C_p}{(r_{ah'})_j}\left(T_s^n - T_g^n\right) + \frac{\rho_{atm}\lambda}{(r_{aw'})_j}\left(q_s^n - q_g^n\right) \\ + \frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl} - z_{1+snl}}\left(T_g^n - T_{2+snl}^n\right) \end{array}\right)\right] \qquad \text{(Eq. 4c)}$$

where $C_{T_s}^4$ is the matrix coefficient in row 4 that is multiplied by $\Delta T_s$, $C_{q_s}^4$ is multiplied by $\Delta q_s$, $C_{(T_v)_j}^4$ is multiplied by $\Delta (T_v)_j$, and $C_{T_g}^4$ is multiplied by $\Delta T_g$. A smoothing filter is introduced by multiplying the time step, $\Delta t$, by a factor of 2. $F_{T_g}$ is the RHS term of Eq. 4.

## 3. The Matrix

Using the LAPACK matrix solver DGESV, CLM solves the set of simultaneous equations described in section 2 once per time step *n* for each column in a land grid cell (except over lake and urban land units where CLM's existing method remains unchanged). A grid cell's lake, wetland, glacier, urban, and soil fraction each occupies a separate column. The unknowns in this set of equations include various near-surface prognostic temperature and humidity variables for model time step *n* + 1:

The canopy air space temperature and humidity, $T_s$ and $q_s$, which represent the column's canopy air space state, $T_g$, the temperature of the top soil/snow layer, and $(T_v)_j$, the vegetation temperature indexed by pft $j$, which does not have values over bare ground.

We write the equations in matrix form $(A \cdot x = B)$ for the sample case of one pft and no bare ground present (*npft* = 1). With more pfts, the number of rows and columns corresponding to $T_v$ would equal the number of pfts, *npft* (minus one when bare ground is present). With only bare ground present (*npft* = 1), the rows and columns corresponding to $T_v$ drop out of the matrix.

$$\begin{pmatrix} C_{T_s}^1 & 0 & C_{(T_v)_j}^1 & C_{T_g}^1 \\ 0 & C_{q_s}^2 & C_{(T_v)_j}^2 & C_{T_g}^2 \\ C_{T_s}^3 & C_{q_s}^3 & C_{(T_v)_j}^3 & C_{T_g}^3 \\ C_{T_s}^4 & C_{q_s}^4 & C_{(T_v)_j}^4 & C_{T_g}^4 \end{pmatrix} \times \begin{bmatrix} \Delta T_s \\ \Delta q_s \\ \Delta (T_v)_j \\ \Delta T_g \end{bmatrix} = \begin{bmatrix} F_{T_s} \\ F_{q_s} \\ F_{(T_v)_j} \\ F_{T_g} \end{bmatrix}$$

The matrix coefficients are indexed at top right by the row number (or equation) that they belong to and at bottom right by the column (or prognostic variable) that they correspond to. CLM adjusts the size of matrix $A$ in every grid cell according to the actual number of pfts. The matrix size can range from 3x3 for a column with no pfts (e.g., wetland, glacier, bare soil; *npft* equals 1 but $L + S$ equals 0 in such columns) up to 7x7 for a column with four non-bare ground pfts.

## 4. Steps Toward Implementation

A fortran routine based on SiB3 subroutine sibslv.F90 was written to fill the coefficients of the matrix of section 3 with realistic data from one time step of a singlepoint CLM simulation. The main routine calls a matrix solver (subroutine dgesv) and writes the solution as though one CLM time step has passed.

The fortran routine was originally tested in one column with one pft and no snow:

1. The heat capacities of vegetation and canopy air space were set to zero to mimic CLM assumptions. The matrix solution appeared reasonable but values were different from CLM output at the same time step.
2. Finite heat capacities were used for vegetation and canopy air space and the results changed mainly above ground as expected.
3. A $2\Delta t$ smoothing filter was used in Eq. 1c to Eq. 4c following the approach found in SiB3. The results changed mainly above ground because the smoothing was not used below ground.
4. The routine was changed to accommodate multiple pfts. $T_s$ and $q_s$ were made column level variables. The results did not change when setting *npft* = 1.
5. Solving for two or more identical pfts (*npft* > 1) gave same answers for each of the pfts as for the single pft in test #4.
6. Vegetation related variables were set to zero to test the matrix for the case of bare ground. The results changed mainly above ground as expected.
7. The routine was generalized to accommodate snow. The results did not change when *snl* was set to zero.
8. As this document was written, a few errors were found in the definitions of some matrix coefficients, so answers changed. However, the new results look just as reasonable as the old.

9. This new matrix solution will be linked to the CLM as a replacement to the original iterative solution. In CLM the matrix dimensions will be determined dynamically for variable numbers of pfts and snow layers to ensure maximum computational efficiency. Extensive tests will be performed with the new and the old codes to demonstrate that the new solution works correctly. Some of the tests described earlier in this section will be repeated. Also conservation tests for mass and energy will be performed.
10. We decided to remove the equations solving for soil/snow temperatures other than $T_g$.

## 5. Necessary Code Changes

List subroutines that were removed, added, or changed. List corresponding sections from Oleson *et al.* (2004) that become obsolete.

Apply the limits recommended by Vidale & Stöckli (2005) (see Eq. B1)?

Change the tridiagonal soil/snow temperature matrix to solve for one less layer.

Talked to Retto (March 15$^{th}$, 2006):

- He sent the code that includes the water and energy limits. These limits are applied before solving the matrix.
- He offered to review this document. I suggested after we finish reviewing it ourselves.

## 6. To Do…

Add or just refer to Keith's figures such as 4.1, 5.1, 5.2, 6.1?

Ian (?) suggested that we compile with ATLAS (?)

**Bibliography**


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Kalnay, E., M. Kanamitsu, 1988: Time schemes for strongly nonlinear damping equations. *Mon. Weather Rev.,* **116**, 1945-1958.

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Add a SiB reference OR Ian's write-up?