The New Prognostic Canopy Air Space Solution in the Community Land Model Version 4 (CLM4)

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Preface

This document describes the update to an analytical method of solving for variables near the land-atmosphere interface that will appear in the Community Land Model version 4 (CLM4). This document is an addendum to the standard CLM technical description (Oleson *et al.* 2004). This work was supported in part by the … program through grant ….

Samuel Levis

Boulder, 1 June 2006

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1. Introduction

The Community Land Model (CLM) solves a set of simultaneous equations once per model time step *n*. The unknowns in this set of equations include near-surface prognostic temperature and humidity variables, as well as soil and snow temperature and moisture variables, all at time step $n + 1$.

The near-surface state responds to the conditions prescribed by an atmospheric data set when the CLM operates in offline mode or simulated by an atmospheric general circulation model (GCM) when the CLM operates in coupled mode. In the latter case, CLM's calculated sensible and latent heat fluxes are passed to the GCM to establish two-way land-atmosphere interactions.

CLM versions prior to version 4 employ an iterative scheme of solving for near-surface temperature and humidity variables (Oleson *et al.* 2004). After the iterative scheme, CLM uses a matrix to solve analytically for the soil and snow temperature and moisture.

Here we document the model update to an analytical method of solving for the near-surface temperature and humidity variables using a matrix solver (Vidale & Stöckli 2005). This update appears in CLM version 4. The analytical solution of this matrix of equations simplifies large sections of CLM's code and facilitates the implementation of water isotope tracers in the model. The new solution also reduces sub-daily instability in the heat fluxes resulting from the iterative solution at times.

The implementation of this new matrix leads to a small change in the implementation of CLM's existing matrix of soil and snow temperature and moisture equations. In particular, the temperature of the top layer of soil (or snow if present) is now solved by the new, prognostic canopy air space matrix. As a result the existing matrix solving for soil and snow temperatures now solves for one less layer.

2. The Equations and their Physical Basis

This section includes subsections numbered by matrix row, where each row corresponds to an equation and each equation includes an unknown. The complete set of equations solves simultaneously for all the unknowns. An example of the matrix appears in section 3. Symbols for all variables are consistent with Oleson *et al.* (2004).

Each equation is presented in three forms: (a) the physical form, (b) a series of forms following algebraic transformations, and (c) the matrix coefficient form. The algebraic transformations assume an "explicit coefficient/implicit temperature" numerical scheme (Kalnay and Kanamitsu 1988). "Explicit coefficient" means that we use the resistance terms (r_{ah}, r_{aw}, r_b) calculated at time step n, while "implicit temperature" means that the variables on the right hand side (RHS) of the equations are from time step n + 1.

In the following subsections we define sensible heat fluxes from the GCM's reference height, from the ground, and from the vegetation to the height of the canopy air space (Oleson *et al.* 2004):

$$H = -\frac{\rho_{atm} C_p}{r_{ah}}\left(\overline{\theta}_{atm} - T_s\right)$$
$$H_g = -\frac{\rho_{atm} C_p}{r_{ah'}}\left(T_s - T_g\right) \qquad \text{(Eq. A)}$$
$$H_v = -\rho_{atm} C_p \frac{L+S}{r_b}\left(T_s - T_v\right)$$

We also define the corresponding latent heat fluxes (Oleson *et al.* 2004):

$$\lambda E = -\frac{\rho_{atm}\lambda}{r_{aw}}(q_{atm} - q_s)$$

$$\lambda E_g = -\frac{\rho_{atm}\lambda}{r_{aw'}}(q_s - q_g) \qquad \text{(Eq. B)}$$

$$\lambda E_v = -\rho_{atm}\lambda\left\{f_{wet}\frac{L+S}{r_b} + \frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b + r_s^{sun}} + \frac{L^{sha}}{r_b + r_s^{sha}}\right)\right\}(q_s - q_{sat}^{T_v})$$

λE_v equals the sum of transpiration, λE_v^t, and canopy evaporation, λE_v^w:

$$\lambda E_v^t = -\rho_{atm}\lambda\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b + r_s^{sun}} + \frac{L^{sha}}{r_b + r_s^{sha}}\right)(q_s - q_{sat}^{T_v})$$

$$\lambda E_v^w = -\rho_{atm}\lambda f_{wet}\frac{L+S}{r_b}(q_s - q_{sat}^{T_v}) \qquad \text{(Eq. C)}$$

Furthermore, we define the following derivatives of λE_g and λE_v^w:

$$\frac{\partial \lambda E_g}{\partial q_s} = -\frac{\rho_{atm}\lambda}{r_{aw'}}$$

$$\frac{\partial \lambda E_g}{\partial T_g} = \frac{\rho_{atm}\lambda}{r_{aw'}}\frac{dq_g}{dT_g}$$

$$\frac{\partial \lambda E_v^w}{\partial q_s} = -\rho_{atm}\lambda f_{wet}\frac{L+S}{r_b} \qquad \text{(Eq. D)}$$

$$\frac{\partial \lambda E_v^w}{\partial T_v} = \rho_{atm}\lambda f_{wet}\frac{L+S}{r_b}\frac{dq_{sat}^{T_v}}{dT_v}$$

Explanations and definitions for all the coefficients and variables included in Eqs. A, B, C, and D are provided in the subsection where they first appear.

2.1 Solving for T_s

Eq. 1 solves for T_s (K), the canopy air space temperature. Eq. 1 states that the change in T_s per time increment Δt (s) between time steps n and $n + 1$ is directly proportional to the sum of sensible heat fluxes H_g, H_v, and H (W m^{-2}) (Vidale & Stöckli 2005). These sensible heat fluxes are calculated respectively from the ground, the vegetation, and the GCM's reference height ($z_{atm,h} \approx 30$ m above the ground) to the height of the canopy air space ($z_{0h} + d$, see Oleson *et al.* (2004)). The first two fluxes are positive into and the third is positive away from the canopy air space:

$$c_s \frac{\Delta T_s}{\Delta t} = H_g^{n+1} + H_v^{n+1} - H^{n+1} \qquad \text{(Eq. 1a)}$$

where c_s is the heat capacity of the canopy air space equal to $\rho_{atm} C_p \Delta z$, ρ_{atm} is the density of atmospheric (moist) air (kg m^{-3}), and C_p is the specific heat capacity of dry air (J kg^{-1} K^{-1}). Δz is the greater of 4 m and the difference between the top and bottom heights of the canopy. If Δz tended to zero, c_s would tend to zero and the prognostic form of Eq. 1a would reduce to the diagnostic expression used in CLM prior to version 4 (Vidale & Stöckli 2005). Eq. 1a′ is shown as a reminder of an assumption that ceases to be true in CLM version 4:

$$\lim_{c_s \to 0} c_s \frac{\Delta T_s}{\Delta t} = 0 \Rightarrow H^{n+1} = H_g^{n+1} + H_v^{n+1} \qquad \text{(Eq. 1a')}$$

Starting from Eq. 1a, we carry the $n + 1$ sensible heat flux terms to the LHS, add the corresponding n terms to both sides of the equation, expand all terms (following Eq. A in Section 2), and rearrange the LHS by variable instead of by time step:

$$c_s \frac{\Delta T_s}{\Delta t} + H^{n+1} - H_g^{n+1} - H_v^{n+1} = 0$$

$$c_s \frac{\Delta T_s}{\Delta t} + H^{n+1} - H^n - H_g^{n+1} + H_g^n - H_v^{n+1} + H_v^n = H_v^n + H_g^n - H^n$$

$$
\begin{aligned}
& c_s \frac{\Delta T_s}{\Delta t} - \frac{\rho_{atm} C_p}{r_{ah}} \left(\overline{\theta}_{atm}^{n+1} - T_s^{n+1} \right) + \frac{\rho_{atm} C_p}{r_{ah}} \left(\overline{\theta}_{atm}^{n} - T_s^{n} \right) \\
& + \frac{\rho_{atm} C_p}{r_{ah'}} \left(T_s^{n+1} - T_g^{n+1} \right) - \frac{\rho_{atm} C_p}{r_{ah'}} \left(T_s^{n} - T_g^{n} \right) \\
& + \rho_{atm} C_p \frac{L+S}{r_b} \left(T_s^{n+1} - T_v^{n+1} \right) - \rho_{atm} C_p \frac{L+S}{r_b} \left(T_s^{n} - T_v^{n} \right) \\
& = -\rho_{atm} C_p \frac{L+S}{r_b} \left(T_s^{n} - T_v^{n} \right) - \frac{\rho_{atm} C_p}{r_{ah'}} \left(T_s^{n} - T_g^{n} \right) + \frac{\rho_{atm} C_p}{r_{ah}} \left(\overline{\theta}_{atm}^{n} - T_s^{n} \right)
\end{aligned}
\qquad \text{(Eq. 1b)}
$$

$$
\begin{aligned}
& \left(\frac{c_s}{\Delta t} + \frac{\rho_{atm} C_p}{r_{ah}} + \rho_{atm} C_p \frac{L+S}{r_b} + \frac{\rho_{atm} C_p}{r_{ah'}} \right) \left(T_s^{n+1} - T_s^{n} \right) - \frac{\rho_{atm} C_p}{r_{ah}} \left(\overline{\theta}_{atm}^{n+1} - \overline{\theta}_{atm}^{n} \right) \\
& - \frac{\rho_{atm} C_p}{r_{ah'}} \left(T_g^{n+1} - T_g^{n} \right) - \rho_{atm} C_p \frac{L+S}{r_b} \left(T_v^{n+1} - T_v^{n} \right) \\
& = -\rho_{atm} C_p \frac{L+S}{r_b} \left(T_s^{n} - T_v^{n} \right) - \frac{\rho_{atm} C_p}{r_{ah'}} \left(T_s^{n} - T_g^{n} \right) + \frac{\rho_{atm} C_p}{r_{ah}} \left(\overline{\theta}_{atm}^{n} - T_s^{n} \right)
\end{aligned}
$$

where T_g and T_v are the ground and leaf temperatures (K), $\overline{\theta}_{atm}$ is the potential temperature (K) at the GCM's reference height, L and S are the exposed leaf and stem area index values (m^2 leaf or stem surface m^{-2} ground), r_{ah} is the aerodynamic resistance to sensible heat transfer (s m^{-1}) between CLM's canopy air space and the GCM's reference height, r_b is the leaf boundary layer resistance (s m^{-1}), and r_{ah}' is the aerodynamic resistance to heat transfer (s m^{-1}) between the ground and the canopy air space. Whether in offline or coupled mode, CLM assumes that a dataset will provide or an AGCM will calculate $\overline{\theta}_{atm}^{n+1}$. Therefore, CLM does not calculate $\overline{\theta}_{atm}^{n+1}$ and assumes instead that $\overline{\theta}_{atm}^{n+1} - \overline{\theta}_{atm}^{n} = 0$ to solve the matrix. The corresponding term in Eq. 1b drops out.

T_s, T_g, and $\overline{\theta}_{atm}$, are column level, while T_v, c_s, and the resistance terms are plant functional type (pft) level variables. Generalizing Eq. 1b to include multiple pfts per column and substituting c_s with $\rho_{atm} C_p \Delta z$ gives:

$$
\begin{aligned}
&\sum_{j=1}^{npft}\left[(wt)_j\left(\frac{\Delta z_j}{\Delta t}+\frac{1}{(r_{ah})_j}+\frac{L_j+S_j}{(r_b)_j}+\frac{1}{(r_{ah'})_j}\right)\rho_{atm}C_p\right]\left(T_s^{n+1}-T_s^n\right)\\
&-\sum_{j=1}^{npft}\left[(wt)_j\frac{\rho_{atm}C_p}{(r_{ah'})_j}\right]\left(T_g^{n+1}-T_g^n\right)\\
&-\sum_{j=1}^{npft}\left[(wt)_j\frac{L_j+S_j}{(r_b)_j}\rho_{atm}C_p\left[(T_v^{n+1})_j-(T_v^n)_j\right]\right] \qquad \text{(Eq. 1b′)}\\
&=\sum_{j=1}^{npft}\left[(wt)_j\left(\begin{array}{l}-\rho_{atm}C_p\dfrac{L_j+S_j}{(r_b)_j}\left(T_s^n-(T_v^n)_j\right)\\ -\dfrac{\rho_{atm}C_p}{(r_{ah'})_j}\left(T_s^n-T_g^n\right)+\dfrac{\rho_{atm}C_p}{(r_{ah})_j}\left(\overline{\theta}_{atm}^n-T_s^n\right)\end{array}\right)\right]
\end{aligned}
$$

where j is the pft index ranging from 1 to $npft$ (the number of pfts present in the column) and $(wt)_j$ is the fraction of the column occupied by pft j, where $\sum_{j=1}^{npft}(wt)_j = 1$. CLM includes bare ground in the same column as the vegetation and gives it a pft index. The fraction of the column with bare ground has $L_j = 0$ and $S_j = 0$.

In matrix coefficient form, Eq. 1b′ becomes:

$$C_{T_s}^1 = \sum_{j=1}^{npft}\left[(wt)_j\left(\frac{\Delta z_j}{2\Delta t}+\frac{1}{(r_{ah})_j}+\frac{L_j+S_j}{(r_b)_j}+\frac{1}{(r_{ah'})_j}\right)\rho_{atm}C_p\right]$$

$$C_{(T_v)_j}^1 = -(wt)_j\frac{L_j+S_j}{(r_b)_j}\rho_{atm}C_p \quad \text{for} \quad j=1,2,\ldots,npft$$

$$C_{T_g}^1 = -\sum_{j=1}^{npft}\left[(wt)_j\frac{\rho_{atm}C_p}{(r_{ah'})_j}\right] \qquad \text{(Eq. 1c)}$$

$$F_{T_s} = \sum_{j=1}^{npft}\left[(wt)_j\left(\begin{array}{l} -\rho_{atm}C_p\dfrac{L_j+S_j}{(r_b)_j}\left(T_s^n-(T_v^n)_j\right) \\ -\dfrac{\rho_{atm}C_p}{(r_{ah'})_j}\left(T_s^n-T_g^n\right)+\dfrac{\rho_{atm}C_p}{(r_{ah})_j}\left(\overline{\theta}_{atm}^n-T_s^n\right)\end{array}\right)\right]$$

where $C_{T_s}^1$ is the matrix coefficient in row 1 that is multiplied by ΔT_s, $C_{(T_v)_j}^1$ is multiplied by $\Delta(T_v)_j$ (for j = 1, 2, …, *npft*), and $C_{T_g}^1$ is multiplied by ΔT_g. A smoothing filter is introduced by multiplying the time step, Δt, by a factor of 2. F_{T_s} is the RHS term of Eq. 1.

2.2 Solving for q_s

Eq. 2 solves for q_s, the specific humidity (kg water vapor kg^{-1} air) of the canopy air space. Eq. 2 states that the change in q_s with respect to time is directly proportional to the sum of latent heat fluxes λE_g, λE_v, and λE (W m^{-2}) (Vidale & Stöckli 2005). These latent heat fluxes are calculated respectively from the ground, the vegetation, and the GCM's reference height ($z_{atm,w} = z_{atm,h}$ (Oleson *et al.* 2004)) to the canopy air space height ($z_{0w} + d = z_{0h} + d$ (Oleson *et al.* 2004)) The first two fluxes are positive into and the third is positive away from the canopy air space:

$$\rho_{atm}\lambda\Delta z\frac{\Delta q_s}{\Delta t} = \lambda E_g^{n+1}+\lambda E_v^{n+1}-\lambda E^{n+1} \qquad \text{(Eq. 2a)}$$

where λ (J kg^{-1}) represents the latent heat of sublimation in λE_g when the water content of the top soil/snow layer consists of all ice and no liquid; λ represents the latent heat of vaporization in all other cases; λ converts the units of Eq. 2a from water vapor flux units (kg m^{-2} s^{-1}) to energy flux units (W m^{-2}). Other terms in Eq. 2a have been defined previously.

If Δz tended to zero, the prognostic form of Eq. 2a would reduce to the diagnostic expression used in CLM prior to version 4 (not shown, but for an example see Eq. 1a′ in section 2.1).

Next we carry the *n* + 1 latent heat flux terms to the LHS, add the corresponding *n* terms to both sides of the equation, expand all terms (refer to Eq. B in Section 2), and rearrange by variable instead of by time step:

$$\rho_{atm}\lambda\Delta z\frac{\Delta q_s}{\Delta t}+\lambda E^{n+1}-\lambda E_g^{n+1}-\lambda E_v^{n+1}=0$$

$$\rho_{atm}\lambda\Delta z\frac{\Delta q_s}{\Delta t}+\lambda E^{n+1}-\lambda E^{n}-\lambda E_g^{n+1}+\lambda E_g^{n}-\lambda E_v^{n+1}+\lambda E_v^{n}=\lambda E_v^{n}+\lambda E_g^{n}-\lambda E^{n}$$

$$\begin{aligned}
&\rho_{atm}\lambda\Delta z\frac{\Delta q_s}{\Delta t}-\frac{\rho_{atm}\lambda}{r_{aw}}\left(q_{atm}^{n+1}-q_s^{n+1}\right)+\frac{\rho_{atm}\lambda}{r_{aw}}\left(q_{atm}^{n}-q_s^{n}\right)+\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^{n+1}-q_g^{n+1}\right)-\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^{n}-q_g^{n}\right)\\
&+\rho_{atm}\lambda\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(q_s^{n+1}-q_{sat}^{T_v^{n+1}}\right)\\
&-\rho_{atm}\lambda\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(q_s^{n}-q_{sat}^{T_v^{n}}\right)\\
&=-\rho_{atm}\lambda\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(q_s^{n}-q_{sat}^{T_v^{n}}\right)\\
&-\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^{n}-q_g^{n}\right)+\frac{\rho_{atm}\lambda}{r_{aw}}\left(q_{atm}^{n}-q_s^{n}\right)
\end{aligned}$$

$$\rho_{atm}\lambda\left\{\frac{\Delta z}{\Delta t}+\frac{1}{r_{aw}}+f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)+\frac{1}{r_{aw'}}\right\}\left(q_s^{n+1}-q_s^n\right)$$
$$-\rho_{atm}\lambda\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(q_{sat}^{T_v^{n+1}}-q_{sat}^{T_v^n}\right)$$
$$-\frac{\rho_{atm}\lambda}{r_{aw}}\left(q_{atm}^{n+1}-q_{atm}^n\right)-\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_g^{n+1}-q_g^n\right) \qquad \text{(Eq. 2b.1)}$$
$$=-\rho_{atm}\lambda\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(q_s^n-q_{sat}^{T_v^n}\right)$$
$$-\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^n-q_g^n\right)+\frac{\rho_{atm}\lambda}{r_{aw}}\left(q_{atm}^n-q_s^n\right)$$

where f_{wet} is the wetted fraction of the canopy (leaves and stems) and f_{dry} is the fraction of leaves that are dry and able to photosynthesize (f_{wet} and f_{dry} are defined mathematically in Oleson *et al.* (2004) and $f_{dry} \neq 1 - f_{wet}$ in general). When the soil moisture function that limits transpiration, β_t (Oleson *et al.* 2004), drops to 1 x 10^{-10} or less, f_{dry} reduces to 0. When dew is present, f_{dry} equals 0 and f_{wet} equals 1. L^{sun} and L^{sha} are the sunlit and shaded components of L (m^2 m^{-2}), r_s^{sun} and r_s^{sha} are the sunlit and shaded stomatal resistances (s m^{-1}), r_{aw} is the aerodynamic resistance to water vapor transfer (s m^{-1}) between the canopy air space and the GCM's reference height, r_{aw}' is the aerodynamic resistance to water vapor transfer (s m^{-1}) between the ground and the canopy air space, q_g is the specific humidity (kg kg^{-1}) at the ground, and $q_{sat}^{T_v}$ is the saturated specific humidity (kg kg^{-1}) at temperature T_v. CLM assumes that a dataset or an AGCM will provide the value of q_{atm}^{n+1}. Therefore, CLM does not calculate q_{atm}^{n+1} and assumes instead that $q_{atm}^{n+1}-q_{atm}^n=0$ to solve the matrix, so the corresponding term in Eq. 2b.1 drops out.

Assuming that $\frac{dq_{sat}^{T}}{dT}=\frac{q_{sat}^{T^{n+1}}-q_{sat}^{T^{n}}}{T^{n+1}-T^{n}}$, where q_{sat}^{T} is the saturated specific humidity at temperature *T*, and assuming that $\frac{dq_g}{dT_g}=\alpha\frac{dq_{sat}^{T_g}}{dT_g}$ given that $q_g=\alpha q_{sat}^{T_g}$, where q_g is the specific humidity at the ground as a function of the saturated specific humidity at the ground (section 5.2 of Oleson et al. (2004)), we substitute the terms $q_g^{n+1}-q_g^{n}$ and $q_{sat}^{T_v^{n+1}}-q_{sat}^{T_v^{n}}$ to get Eq. 2b.2:

$$
\begin{aligned}
&\rho_{atm}\lambda\left\{\frac{\Delta z}{\Delta t}+\frac{1}{r_{aw}}+f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)+\frac{1}{r_{aw'}}\right\}\left(q_s^{n+1}-q_s^{n}\right)\\
&-\rho_{atm}\lambda\frac{dq_{sat}^{T_v}}{dT_v}\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(T_v^{n+1}-T_v^{n}\right)\\
&-\frac{\rho_{atm}\lambda}{r_{aw'}}\frac{dq_g}{dT_g}\left(T_g^{n+1}-T_g^{n}\right) \qquad \text{(Eq. 2b.2)}\\
&=-\rho_{atm}\lambda\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(q_s^{n}-q_{sat}^{T_v^{n}}\right)\\
&-\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^{n}-q_g^{n}\right)+\frac{\rho_{atm}\lambda}{r_{aw}}\left(q_{atm}^{n}-q_s^{n}\right)
\end{aligned}
$$

Eq. D in the introductory part of Section 2 defines the terms $\frac{\partial\lambda E_g}{\partial q_s},\frac{\partial\lambda E_g}{\partial T_g},\frac{\partial\lambda E_v^{w}}{\partial q_s}$, and $\frac{\partial\lambda E_v^{w}}{\partial T_v}$, all of which appear in expanded form in the LHS of Eq. 2b.2. These terms represent the rate of change of λE_g or λE_v^{w} with respect to q_s or *T*. We focus on these four terms in particular because we need to make sure that:

$$
\begin{aligned}
-\frac{\partial \lambda E_g}{\partial q_s} &= \frac{\rho_{atm}\lambda}{r_{aw'}} \le \frac{\alpha\lambda\left(w_{liq,snl+1} + w_{ice,snl+1}\right)}{\Delta q_s^{\max}\Delta t} \\
\frac{\partial \lambda E_g}{\partial T_g} &= \frac{\rho_{atm}\lambda}{r_{aw'}}\frac{dq_g}{dT_g} \le \frac{\alpha\lambda\left(w_{liq,snl+1} + w_{ice,snl+1}\right)}{\Delta T_g^{\max}\Delta t} \\
-\frac{\partial \lambda E_v^w}{\partial q_s} &= \rho_{atm}\lambda f_{wet}\frac{L+S}{r_b} \le \frac{\alpha\lambda W_{can}}{\Delta q_s^{\max}\Delta t} \\
\frac{\partial \lambda E_v^w}{\partial T_v} &= \rho_{atm}\lambda f_{wet}\frac{L+S}{r_b}\frac{dq_{sat}^{T_v}}{dT_v} \le \frac{\alpha\lambda W_{can}}{\Delta T_v^{\max}\Delta t}
\end{aligned}
\qquad \text{(Eq. 2b.3)}
$$

where α is the "security constant" equal to 0.75 (Vidale & Stöckli 2005), $w_{liq,snl+1}$ and $w_{ice,snl+1}$ are the liquid and solid water in the top soil or snow (if present) layer (kg m^{-2}), W_{can} is the amount of water on the canopy per unit area of ground (kg m^{-2}), and $\Delta T_v^{\max}$ is the maximum allowed T_v increment equal to 3K per time step. Finally $\Delta q_s^{\max} \approx \Delta e_s^{\max}\frac{\varepsilon}{P_{atm}}$ (Iribarne & Godson 1989) where P_{atm} is the atmospheric pressure (Pa), ε is the ratio of the molecular weights of water and dry air equal to 0.622, and $\Delta e_s^{\max}$ is the maximum allowed vapor pressure increment equal to 0.05 Pa per time step. The limits in Eq. 2b.3 ensure that evaporation terms from the canopy and ground do not exceed the water available on the canopy and in the top soil/snow layer. A corresponding limit could be applied to transpiration, but this flux's coefficient f_{dry} already responds to the soil moisture function that limits transpiration, β_t.

For clarity we rewrite Eq. 2b.2 with the substitutions from Eq. 2b.3:

$$
\begin{aligned}
&\left[-\frac{\partial \lambda E_g}{\partial q_s} - \frac{\partial \lambda E_v^w}{\partial q_s} + \rho_{atm}\lambda\left\{\frac{\Delta z}{\Delta t} + \frac{1}{r_{aw}} + \frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b + r_s^{sun}} + \frac{L^{sha}}{r_b + r_s^{sha}}\right)\right\}\right]\left(q_s^{n+1} - q_s^n\right) \\
&-\left[\frac{\partial \lambda E_v^w}{\partial T_v} + \rho_{atm}\lambda\frac{dq_{sat}^{T_v}}{dT_v}\left\{\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b + r_s^{sun}} + \frac{L^{sha}}{r_b + r_s^{sha}}\right)\right\}\right]\left(T_v^{n+1} - T_v^n\right) \\
&-\frac{\partial \lambda E_g}{\partial T_g}\left(T_g^{n+1} - T_g^n\right) \qquad \text{(Eq. 2b.4)} \\
&= -\rho_{atm}\lambda\left\{f_{wet}\frac{L+S}{r_b} + \frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b + r_s^{sun}} + \frac{L^{sha}}{r_b + r_s^{sha}}\right)\right\}\left(q_s^n - q_{sat}^{T_v^n}\right) \\
&-\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^n - q_g^n\right) + \frac{\rho_{atm}\lambda}{r_{aw}}\left(q_{atm}^n - q_s^n\right)
\end{aligned}
$$

I will make the remaining substitutions after Monday's meeting to ensure that all agree with what I am doing. Keith pointed me to p. 76 of his tech note for clm's solution to multiple pfts per soil column. Gordon suggested that I remove the soil water limit altogether because the water could always come from the next soil layer, and at the next time step alpha would equal 0, so E_g would also go to 0.

Regarding the question whether $\alpha<1$ allows canopy water to ever dry up completely, I will ask Reto in Breckenridge.

In this section I modified the equation numbering system a bit. Do people prefer that I use this numbering scheme throughout the document?

T_g, q_s, and q_{atm}, are column level, while T_v and the resistance terms are pft level variables. Generalizing Eq. 2b.2 to include multiple pfts per column gives:

$$\sum_{j=1}^{npft}\left[(wt)_j \rho_{atm}\lambda\left\{\begin{array}{l}\frac{\Delta z_j}{\Delta t}+\frac{1}{(r_{aw})_j}+\frac{1}{(r_{aw'})_j}+(f_{wet})_j\frac{L_j+S_j}{(r_b)_j}\\+\frac{(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j}+\frac{L_j^{sha}}{(r_b)_j+(r_s^{sha})_j}\right)\end{array}\right\}\right]\left(q_s^{n+1}-q_s^n\right)$$

$$-\sum_{j=1}^{npft}\left[(wt)_j \rho_{atm}\lambda\frac{dq_{sat}^{(T_v)_j}}{d(T_v)_j}\left\{\begin{array}{l}(f_{wet})_j\frac{L_j+S_j}{(r_b)_j}\\+\frac{(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j}+\frac{L_j^{sha}}{(r_b)_j+(r_s^{sha})_j}\right)\end{array}\right\}\left[(T_v^{n+1})_j-(T_v^n)_j\right]\right]$$

$$-\sum_{j=1}^{npft}\left[(wt)_j\frac{\rho_{atm}\lambda}{(r_{aw'})_j}\frac{dq_g}{dT_g}\right]\left(T_g^{n+1}-T_g^n\right) \qquad \text{(Eq.2b.5)}$$

$$=\sum_{j=1}^{npft}\left[(wt)_j\left(\begin{array}{l}-\rho_{atm}\lambda\left\{(f_{wet})_j\frac{L_j+S_j}{(r_b)_j}+\frac{(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j}+\frac{L_j^{sha}}{(r_b)_j+(r_s^{sha})_j}\right)\right\}\left(q_s^n-q_{sat}^{(T_v^n)_j}\right)\\-\frac{\rho_{atm}\lambda}{(r_{aw'})_j}\left(q_s^n-q_g^n\right)+\frac{\rho_{atm}\lambda}{(r_{aw})_j}\left(q_{atm}^n-q_s^n\right)\end{array}\right)\right]$$

In matrix coefficient form, Eq. 2b.4 becomes:

$$C_{q_s}^2=\sum_{j=1}^{npft}\left[(wt)_j \rho_{atm}\lambda\left\{\begin{array}{l}\frac{\Delta z_j}{2\Delta t}+\frac{1}{(r_{aw})_j}+\frac{1}{(r_{aw'})_j}+(f_{wet})_j\frac{L_j+S_j}{(r_b)_j}\\+\frac{(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j}+\frac{L_j^{sha}}{(r_b)_j+(r_s^{sha})_j}\right)\end{array}\right\}\right]$$

$$C_{(T_v)_j}^2=-(wt)_j \rho_{atm}\lambda\frac{dq_{sat}^{(T_v)_j}}{d(T_v)_j}\left\{\begin{array}{l}(f_{wet})_j\frac{L_j+S_j}{(r_b)_j}\\+\frac{(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j}+\frac{L_j^{sha}}{(r_b)+(r_s^{sha})_j}\right)\end{array}\right\} \text{ for } j=1,2,\ldots,npft$$

$$C_{T_g}^2=-\sum_{j=1}^{npft}\left[(wt)_j\frac{\rho_{atm}\lambda}{(r_{aw'})_j}\frac{dq_g}{dT_g}\right] \qquad \text{(Eq.2c)}$$

$$F_{q_s}=\sum_{j=1}^{npft}\left[(wt)_j\left(\begin{array}{l}-\rho_{atm}\lambda\left\{(f_{wet})_j\frac{L_j+S_j}{(r_b)_j}+\frac{(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j}+\frac{L_j^{sha}}{(r_b)_j+(r_s^{sha})_j}\right)\right\}\left(q_s^n-q_{sat}^{(T_v^n)_j}\right)\\-\frac{\rho_{atm}\lambda}{(r_{aw'})_j}\left(q_s^n-q_g^n\right)+\frac{\rho_{atm}\lambda}{(r_{aw})_j}\left(q_{atm}^n-q_s^n\right)\end{array}\right)\right]$$

where $C^2_{q_s}$ is the matrix coefficient in row 2 that is multiplied by Δq_s, $C^2_{(T_v)_j}$ is multiplied by $\Delta(T_v)_j$, and $C^2_{T_g}$ is multiplied by ΔT_g. A smoothing filter is introduced by multiplying the time step, Δt, by a factor of 2. F_{q_s} is the RHS term of Eq. 2. Although λ could cancel out of all the terms in Eq. 2 as latent heat of vaporization, λ could also represent the latent heat of sublimation in $C^2_{T_g}$ if the top soil/snow layer's moisture were all ice. Therefore, we keep λ in all the terms of Eq. 2.

2.3 Solving for T_v

Eq. 3 states that vegetation temperature, T_v (K), changes with time as a function of the net energy available to the vegetation (W m^{-2}), accounting for radiation and heat flux terms, as well as for changes in the vegetation's net longwave radiation. The radiation terms include the vegetation-absorbed net solar, $\vec{S}_v$ (positive into vegetation), and net longwave radiation, $\vec{L}_v$ (positive away from vegetation). The heat flux terms include the sensible and latent heat fluxes, H_v and λE_v (positive away from vegetation). The change in the vegetation's net longwave radiation from time step n to $n + 1$ with respect to temperature is given by $\left.\frac{d\vec{L}_v}{dT_g}\right|_n \Delta T_g \delta_{veg} + \left.\frac{d\vec{L}_v}{dT_v}\right|_n \Delta T_v \delta_{veg}$ (positive away from vegetation).

$$c_v \frac{\Delta T_v}{\Delta t} = \vec{S}^n_v - \vec{L}^n_v - H^{n+1}_v - \lambda E^{n+1}_v - \left.\frac{d\vec{L}_v}{dT_g}\right|_n \Delta T_g \delta_{veg} - \left.\frac{d\vec{L}_v}{dT_v}\right|_n \Delta T_v \delta_{veg} \qquad \text{(Eq. 3a)}$$

where c_v (J m^{-2} K^{-1}) is the heat capacity of the vegetation equal to $(L+S)C_{liq}W_{l+s} + C_{liq}W_{can}$, where C_{liq} is the specific heat capacity of water (J kg^{-1} K^{-1}), W_{l+s} is the amount of water in leaves and stems set to 0.2 kg m^{-2} leaf and stem area, W_{can}

is the amount of water on the canopy per unit area of ground (kg m^{-2}), and δ_{veg} is a step function equal to zero for $L + S < 0.05$ and equal to one otherwise.

As in previous sections, now we transform Eq. 3a to Eq. 3b. At this time we also replace $\left.\frac{d\vec{L}_v}{dT_g}\right|_n$ and $\left.\frac{d\vec{L}_v}{dT_v}\right|_n$ with $-4\varepsilon_v\varepsilon_g\sigma\left(T_g^n\right)^3$ and $4\left[2-\varepsilon_v\left(1-\varepsilon_g\right)\right]\varepsilon_v\sigma\left(T_v^n\right)^3$, respectively:

$$c_v\frac{\Delta T_v}{\Delta t}+H_v^{n+1}-H_v^n+\lambda E_v^{n+1}-\lambda E_v^n+\left.\frac{d\vec{L}_v}{dT_g}\right|_n\Delta T_g\delta_{veg}+\left.\frac{d\vec{L}_v}{dT_v}\right|_n\Delta T_v\delta_{veg}=\vec{S}_v^n-\vec{L}_v^n-H_v^n-\lambda E_v^n$$

$$\begin{aligned}
&\frac{c_v}{\Delta t}\left(T_v^{n+1}-T_v^n\right)-\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^{n+1}-T_v^{n+1}\right)+\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^n-T_v^n\right)\\
&-\rho_{atm}\lambda\left[f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_s^{n+1}-q_{sat}^{T_v^{n+1}}\right)\\
&+\rho_{atm}\lambda\left[f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_s^n-q_{sat}^{T_v^n}\right)\\
&+4\left[2-\varepsilon_v\left(1-\varepsilon_g\right)\right]\varepsilon_v\sigma\left(T_v^n\right)^3\delta_{veg}\left(T_v^{n+1}-T_v^n\right)-4\varepsilon_v\varepsilon_g\sigma\left(T_g^n\right)^3\delta_{veg}\left(T_g^{n+1}-T_g^n\right)\\
&=\vec{S}_v^n-\vec{L}_v^n+\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^n-T_v^n\right)+\rho_{atm}\lambda\left[f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_s^n-q_{sat}^{T_v^n}\right)
\end{aligned}$$

$$\begin{aligned}
&\left[\frac{c_v}{\Delta t}+4\left[2-\varepsilon_v\left(1-\varepsilon_g\right)\right]\varepsilon_v\sigma\left(T_v^n\right)^3\delta_{veg}+\rho_{atm}C_p\frac{L+S}{r_b}\right]\left(T_v^{n+1}-T_v^n\right)-\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^{n+1}-T_s^n\right)\\
&+\rho_{atm}\lambda\left[f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_{sat}^{T_v^{n+1}}-q_{sat}^{T_v^n}\right)\\
&-\rho_{atm}\lambda\left[f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_s^{n+1}-q_s^n\right)\\
&-4\varepsilon_v\varepsilon_g\sigma\left(T_g^n\right)^3\delta_{veg}\left(T_g^{n+1}-T_g^n\right)\\
&=\vec{S}_v^n-\vec{L}_v^n+\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^n-T_v^n\right)+\rho_{atm}\lambda\left[f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_s^n-q_{sat}^{T_v^n}\right)
\end{aligned} \qquad \text{(Eq. 3b)}$$

where ε_v and ε_g are the vegetation and ground emissivities, and σ is the Stefan-Boltzmann constant (W m^{-2} K^{-4}).

As done in Eq. 2b, we next substitute $q_{sat}^{T_v^{n+1}} - q_{sat}^{T_v^n}$ with $\frac{dq_{sat}^{T_v}}{dT_v}\left(T_v^{n+1} - T_v^n\right)$:

$$\begin{aligned}
&\left[\frac{c_v}{\Delta t} + 4\left[2 - \varepsilon_v\left(1 - \varepsilon_g\right)\right]\varepsilon_v\sigma\left(T_v^n\right)^3\delta_{veg} + \rho_{atm}C_p\frac{L+S}{r_b}\right. \\
&\left. + \rho_{atm}\lambda\frac{dq_{sat}^{T_v}}{dT_v}\left\{f_{wet}\frac{L+S}{r_b} + \frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b + r_s^{sun}} + \frac{L^{sha}}{r_b + r_s^{sha}}\right)\right\}\right]\left(T_v^{n+1} - T_v^n\right) \\
&- \rho_{atm}\lambda\left[f_{wet}\frac{L+S}{r_b} + \frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b + r_s^{sun}} + \frac{L^{sha}}{r_b + r_s^{sha}}\right)\right]\left(q_s^{n+1} - q_s^n\right) \qquad \text{(Eq. 3b')} \\
&- \rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^{n+1} - T_s^n\right) - 4\varepsilon_v\varepsilon_g\sigma\left(T_g^n\right)^3\delta_{veg}\left(T_g^{n+1} - T_g^n\right) \\
&= \vec{S}_v^n - \vec{L}_v^n + \rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^n - T_v^n\right) + \rho_{atm}\lambda\left[f_{wet}\frac{L+S}{r_b} + \frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b + r_s^{sun}} + \frac{L^{sha}}{r_b + r_s^{sha}}\right)\right]\left(q_s^n - q_{sat}^{T_v^n}\right)
\end{aligned}$$

T_s, q_s, and T_g are column level, while T_v and the resistance terms are pft level variables. Generalizing Eq. 3b′ to include multiple pfts per column gives:

$$\begin{aligned}
&\left[\frac{(c_v)_j}{\Delta t} + 4\left[2 - (\varepsilon_v)_j\left(1 - \varepsilon_g\right)\right](\varepsilon_v)_j\sigma\left(T_v^n\right)_j^3(\delta_{veg})_j + \rho_{atm}C_p\frac{L_j+S_j}{(r_b)_j}\right. \\
&\left. + \rho_{atm}\lambda\frac{dq_{sat}^{(T_v)_j}}{d(T_v)_j}\left\{(f_{wet})_j\frac{L_j+S_j}{(r_b)_j} + \frac{(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j + (r_s^{sha})_j}\right)\right\}\right]\left[(T_v^{n+1})_j - (T_v^n)_j\right] \\
&- \rho_{atm}\lambda\left[(f_{wet})_j\frac{L_j+S_j}{(r_b)_j} + \frac{(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j + (r_s^{sha})_j}\right)\right]\left(q_s^{n+1} - q_s^n\right) \\
&- \rho_{atm}C_p\frac{L_j+S_j}{(r_b)_j}\left(T_s^{n+1} - T_s^n\right) - 4(\varepsilon_v)_j\varepsilon_g\sigma\left(T_g^n\right)^3(\delta_{veg})_j\left(T_g^{n+1} - T_g^n\right) \qquad \text{(Eq. 3b'')} \\
&= (\vec{S}_v^n)_j - (\vec{L}_v^n)_j + \rho_{atm}C_p\frac{L_j+S_j}{(r_b)_j}\left(T_s^n - (T_v^n)_j\right) \\
&+ \rho_{atm}\lambda\left[(f_{wet})_j\frac{L_j+S_j}{(r_b)_j} + \frac{(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j + (r_s^{sha})_j}\right)\right]\left(q_s^n - q_{sat}^{(T_v^n)_j}\right)
\end{aligned}$$

for $j = 1, 2, \ldots, npft$. There are as many equations solving for T_v as pfts in the soil column. For bare ground, all terms in this equation reduce to zero and the equation is omitted from the matrix.

In matrix coefficient form, Eq. 3b″ becomes:

$$C_{T_s}^{2+j} = -\rho_{atm} C_p \frac{L_j + S_j}{(r_b)_j}$$

$$C_{q_s}^{2+j} = -\rho_{atm} \lambda \left\{ (f_{wet})_j \frac{L_j + S_j}{(r_b)_j} + \frac{(f_{dry})_j}{L_j} \left(\frac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j + (r_s^{sha})_j} \right) \right\}$$

$$C_{(T_v)_j}^{2+j} = \frac{(c_v)_j}{2\Delta t} + 4\left[2 - (\varepsilon_v)_j (1 - \varepsilon_g)\right](\varepsilon_v)_j \sigma \left(T_v^n\right)_j^3 (\delta_{veg})_j + \rho_{atm} C_p \frac{L_j + S_j}{(r_b)_j} \qquad \text{(Eq. 3c)}$$

$$+ \rho_{atm} \lambda \frac{dq_{sat}^{(T_v)_j}}{d(T_v)_j} \left\{ (f_{wet})_j \frac{L_j + S_j}{(r_b)_j} + \frac{(f_{dry})_j}{L_j} \left(\frac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j + (r_s^{sha})_j} \right) \right\}$$

$$C_{T_g}^{2+j} = -4(\varepsilon_v)_j \varepsilon_g \sigma \left(T_g^n\right)^3 (\delta_{veg})_j$$

$$F_{(T_v)_j} \Big|_{j=1}^{npft} = (\vec{S}_v^n)_j - (\vec{L}_v^n)_j + \rho_{atm} C_p \frac{L_j + S_j}{(r_b)_j} \left(T_s^n - (T_v^n)_j \right)$$

$$+ \rho_{atm} \lambda \left[(f_{wet})_j \frac{L_j + S_j}{(r_b)_j} + \frac{(f_{dry})_j}{L_j} \left(\frac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j + (r_s^{sha})_j} \right) \right] \left(q_s^n - q_{sat}^{(T_v^n)_j} \right)$$

for $j = 1, 2, \ldots, npft$. $C_{T_s}^{2+j}$ is the matrix coefficient in row $2 + j$ that is multiplied by ΔT_s, $C_{q_s}^{2+j}$ is multiplied by Δq_s, $C_{(T_v)_j}^{2+j}$ is multiplied by $\Delta (T_v)_j$, and $C_{T_g}^{2+j}$ is multiplied by ΔT_g. A smoothing filter is introduced by multiplying the time step, Δt, by a factor of 2. $F_{(T_v)_j}$ is the RHS term of Eq. 3. The limits in Eq. 2b.3 also apply to Eq. 3 so as to ensure that evaporation from the canopy cannot exceed the water available on the canopy.

The matrix solves for $(T_v)_j$ for all the pfts present in a soil column. To continue numbering the subsections of section 2 so that they correspond to matrix row numbers, we assume here the absence of bare ground and the presence of one pft ($npft$ = 1).

Therefore, this section (2.3) discussed matrix row 3 and the following section (2.4) will discuss matrix row 4.

2.4 Solving for T_g

In CLM, ground properties correspond to properties of the soil or snow layer that is in contact with the atmosphere. Eq. 4 solves for T_g, the ground temperature (K), stating that $\frac{\Delta T_g}{\Delta t}$ is a function of the net energy available to the top soil/snow layer (W m^{-2}). As in Eq. 3, net energy includes radiation and heat flux terms, as well as changes in net longwave radiation at the ground. The radiation terms include ground-absorbed net solar, $\vec{S}_g$ (positive into the top soil/snow layer), and net longwave radiation, $\vec{L}_g$ (positive away from top soil/snow layer). The heat flux terms include sensible, latent, and soil/snow heat fluxes, H_g, λE_g, and F_{1+snl} (positive away from top soil/snow layer). F_{1+snl}, the lower boundary condition of this matrix, becomes the upper boundary condition of the soil/snow temperature matrix later in the same time step, and Oleson *et al.* (2004) refer to this flux as G. The ground's net longwave radiation changes with respect to temperature as $\left.\frac{d\vec{L}_g}{dT_g}\right|_n \Delta T_g + \left.\frac{d\vec{L}_g}{dT_v}\right|_n \Delta T_v$ (positive away from top soil/snow layer).

$$c_{1+snl}\Delta z_{i*}\frac{\Delta T_g}{\Delta t} = \vec{S}_g^n - \vec{L}_g^n - H_g^{n+1} - \lambda E_g^{n+1} - F_{1+snl}^{n+1} - \left.\frac{d\vec{L}_g}{dT_g}\right|_n \Delta T_g - \left.\frac{d\vec{L}_g}{dT_v}\right|_n \Delta T_v \qquad \text{(Eq. 4a)}$$

where c_{1+snl} (J m^{-3} K^{-1}) is the volumetric heat capacity of the top soil/snow layer (index 1+*snl*) and *snl* is the number of snow layers ranging from 0 to –5. With no snow the index for the top soil layer is 1, while with five layers of snow the index for the top snow layer is –4. Δz_{i*} (m) is the top soil/snow layer thickness (Eq. 6.29 in Oleson *et al.* (2004)) indexed differently to indicate a numerical adjustment particular to the top layer. This adjustment intends to lower the heat capacity of the top layer to justify clm's assumption that T_g and T_1, the ground and top layer temperatures are one and the same.

Transformations similar to the ones used in sections 2.2 and 2.3 lead from Eq. 4a to Eq. 4b. Here $\left.\frac{d\vec{L}_g}{dT_g}\right|_n$ and $\left.\frac{d\vec{L}_g}{dT_v}\right|_n$ are replaced with $4\varepsilon_g\sigma\left(T_g^n\right)^3$ and $-4\varepsilon_v\varepsilon_g\delta_{veg}\sigma\left(T_v^n\right)^3$, respectively. Also $q_g^{n+1}-q_g^n$ is replaced with $\frac{dq_g}{dT_g}\left(T_g^{n+1}-T_g^n\right)$, assuming that $\frac{dq_g}{dT_g}=\alpha\frac{dq_{sat}^{T_g}}{dT_g}$ as in Eq. 2b′. Note that T_{2+snl} is known for time step n because the soil/snow temperature matrix is solved separately from the prognostic canopy air space matrix later in the same time step. Therefore, F_{1+snl}^{n+1} is defined as $-\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\left(T_g^{n+1}-T_{2+snl}^n\right)$:

$$c_{1+snl}\Delta z_{i*}\frac{\Delta T_g}{\Delta t}+\left.\frac{d\vec{L}_g}{dT_g}\right|_n\Delta T_g+\left.\frac{d\vec{L}_g}{dT_v}\right|_n\Delta T_v$$
$$+H_g^{n+1}-H_g^n+\lambda E_g^{n+1}-\lambda E_g^n+F_{1+snl}^{n+1}-F_{1+snl}^n=\vec{S}_g^n-\vec{L}_g^n-H_g^n-\lambda E_g^n-F_{1+snl}^n$$

$$c_{1+snl}\Delta z_{i*}\frac{\Delta T_g}{\Delta t}+4\varepsilon_g\sigma\left(T_g^n\right)^3\left(T_g^{n+1}-T_g^n\right)-4\varepsilon_v\varepsilon_g\sigma\left(T_v^n\right)^3\delta_{veg}\left(T_v^{n+1}-T_v^n\right)$$
$$-\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^{n+1}-T_g^{n+1}\right)+\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^n-T_g^n\right)-\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^{n+1}-q_g^{n+1}\right)+\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^n-q_g^n\right)$$
$$-\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\left(T_g^{n+1}-T_{2+snl}^n\right)+\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\left(T_g^n-T_{2+snl}^n\right)$$
$$=\vec{S}_g^n-\vec{L}_g^n+\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^n-T_g^n\right)+\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^n-q_g^n\right)+\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\left(T_g^n-T_{2+snl}^n\right)$$

$$
\begin{aligned}
&c_{1+snl}\Delta z_{i*}\frac{\Delta T_g}{\Delta t}+\left[\frac{\rho_{atm}C_p}{r_{ah'}}+4\varepsilon_g\sigma\left(T_g^n\right)^3-\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\right]\left(T_g^{n+1}-T_g^n\right)\\
&-\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^{n+1}-T_s^n\right)-\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^{n+1}-q_s^n\right)+\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_g^{n+1}-q_g^n\right)\\
&-4\varepsilon_v\varepsilon_g\sigma\left(T_v^n\right)^3\delta_{veg}\left(T_v^{n+1}-T_v^n\right)\\
&=\vec{S}_g^n-\vec{L}_g^n+\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^n-T_g^n\right)+\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^n-q_g^n\right)+\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\left(T_g^n-T_{2+snl}^n\right)
\end{aligned}
\qquad \text{(Eq. 4b)}
$$

$$
\begin{aligned}
&\left[\frac{c_{1+snl}\Delta z_{i*}}{\Delta t}+\frac{\rho_{atm}C_p}{r_{ah'}}+\frac{\rho_{atm}\lambda}{r_{aw'}}\frac{dq_g}{dT_g}+4\varepsilon_g\sigma\left(T_g^n\right)^3-\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\right]\left(T_g^{n+1}-T_g^n\right)\\
&-4\varepsilon_v\varepsilon_g\sigma\left(T_v^n\right)^3\delta_{veg}\left(T_v^{n+1}-T_v^n\right)-\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^{n+1}-T_s^n\right)-\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^{n+1}-q_s^n\right)\\
&=\vec{S}_g^n-\vec{L}_g^n+\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^n-T_g^n\right)+\frac{\rho_{atm}\lambda}{r_{aw'}}\left(q_s^n-q_g^n\right)+\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\left(T_g^n-T_{2+snl}^n\right)
\end{aligned}
$$

where the coefficient λ was defined in section 2.2, while $\lambda[z_{h,1+snl}]$ (W m^{-1} K^{-1}) is the thermal conductivity at the interface between the top and second soil/snow layers, $z_{h,1+snl}$ (m) refers to the depth of that interface, while z_{1+snl} and z_{2+snl} (m) are the depths of the top and second from the top soil/snow layers, respectively.

T_s, q_s, and T_g are column level, while T_v and the resistance terms are pft level variables. Generalizing Eq. 4b for multiple pfts per column gives:

$$\sum_{j=1}^{npft}\left[(wt)_j\left\{\frac{c_{1+snl}\Delta z_{i*}}{\Delta t}+\frac{\rho_{atm}C_p}{(r_{ah'})_j}+\frac{\rho_{atm}\lambda}{(r_{aw'})_j}\frac{dq_g}{dT_g}+4\varepsilon_g\sigma\left(T_g^n\right)^3-\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\right\}\right]\left(T_g^{n+1}-T_g^n\right)$$

$$-\sum_{j=1}^{npft}\left[(wt)_j 4\varepsilon_v\varepsilon_g\sigma\left(T_v^n\right)_j^3(\delta_{veg})_j\left[(T_v^{n+1})_j-(T_v^n)_j\right]\right]$$

$$-\sum_{j=1}^{npft}\left[(wt)_j\frac{\rho_{atm}C_p}{(r_{ah'})_j}\right]\left(T_s^{n+1}-T_s^n\right)-\sum_{j=1}^{npft}\left[(wt)_j\frac{\rho_{atm}\lambda}{(r_{aw'})_j}\right]\left(q_s^{n+1}-q_s^n\right) \qquad \text{(Eq. 4b}'\text{)}$$

$$=\sum_{j=1}^{npft}\left[(wt)_j\left(\begin{array}{l}\vec{S}_g^n-\vec{L}_g^n+\frac{\rho_{atm}C_p}{(r_{ah'})_j}\left(T_s^n-T_g^n\right)+\frac{\rho_{atm}\lambda}{(r_{aw'})_j}\left(q_s^n-q_g^n\right)\\ +\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\left(T_g^n-T_{2+snl}^n\right)\end{array}\right)\right]$$

In matrix coefficient form, Eq. 4b′ becomes:

$$C_{T_s}^4=-\sum_{j=1}^{npft}\left[(wt)_j\frac{\rho_{atm}C_p}{(r_{ah'})_j}\right]$$

$$C_{q_s}^4=-\sum_{j=1}^{npft}\left[(wt)_j\frac{\rho_{atm}\lambda}{(r_{aw'})_j}\right]$$

$$C_{(T_v)_j}^4\Big|_{j=1}^{npft}=-(wt)_j 4\varepsilon_v\varepsilon_g\sigma\left(T_v^n\right)_j^3(\delta_{veg})_j \quad \text{for} \quad j=1,2,\ldots,npft$$

$$C_{T_g}^4=\sum_{j=1}^{npft}\left[(wt)_j\left\{\frac{c_{1+snl}\Delta z_{i*}}{2\Delta t}+\frac{\rho_{atm}C_p}{(r_{ah'})_j}+\frac{\rho_{atm}\lambda}{(r_{aw'})_j}\frac{dq_g}{dT_g}+4\varepsilon_g\sigma\left(T_g^n\right)^3-\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\right\}\right]$$

$$F_{T_g}=\sum_{j=1}^{npft}\left[(wt)_j\left(\begin{array}{l}\vec{S}_g^n-\vec{L}_g^n+\frac{\rho_{atm}C_p}{(r_{ah'})_j}\left(T_s^n-T_g^n\right)+\frac{\rho_{atm}\lambda}{(r_{aw'})_j}\left(q_s^n-q_g^n\right)\\ +\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\left(T_g^n-T_{2+snl}^n\right)\end{array}\right)\right] \qquad \text{(Eq. 4c)}$$

where $C_{T_s}^4$ is the matrix coefficient in row 4 that is multiplied by ΔT_s, $C_{q_s}^4$ is multiplied by Δq_s, $C_{(T_v)_j}^4$ is multiplied by $\Delta(T_v)_j$, and $C_{T_g}^4$ is multiplied by ΔT_g. A smoothing filter is introduced by multiplying the time step, Δt, by a factor of 2. F_{T_g} is the RHS term of Eq. 4. The limits in Eq. 2b.3 also apply to Eq. 4 to ensure that evaporation from the ground cannot exceed the water available in the top soil/snow layer.

Later in the same time step, a tridiagonal matrix solves for the soil and snow temperatures of deeper layers (Oleson *et al.* 2004). Subsequently CLM updates the soil and snow temperatures of all layers (including T_g) to account for the effect of soil/snow water phase changes. In CLM versions prior to version 4, this temperature adjustment led to an adjustment of the sensible and latent heat fluxes, which in CLM4 we will neglect for simplicity. In every time step, we keep track of two T_g values: one which is consistent with the state of the canopy air space and one which is adjusted for the soil/snow water phase changes.

3. The Matrix

Using the LAPACK matrix solver DGESV, CLM solves the set of simultaneous equations described in section 2 once per time step n for each column in a land grid cell. A grid cell's lake, wetland, glacier, urban, and soil fraction each occupies a separate land unit, each with one column in the current version of CLM. The matrix was not implemented in the lake and urban land units, so CLM uses the existing iterative method there. The wetland and glacier land units are treated as bare ground for the purposes of this matrix.

The unknowns in this set of equations include various near-surface prognostic temperature and humidity variables for model time step $n + 1$: Canopy air space temperature and humidity, T_s and q_s, which represent the column's canopy air space state, T_g, the temperature of the top soil/snow layer, and $(T_v)_j$, the vegetation temperature indexed by pft j, which remains undefined over bare ground.

We write the equations in matrix form $(A \cdot x = B)$ for the sample case of one pft and no bare ground present (*npft* = 1). With more pfts, the number of rows and columns corresponding to T_v would equal the number of pfts, *npft* (minus one when bare ground is present). With only bare ground present (*npft* = 1), the rows and columns corresponding to T_v drop out of the matrix.

$$\begin{pmatrix} C^1_{T_s} & 0 & C^1_{(T_v)_j} & C^1_{T_g} \\ 0 & C^2_{q_s} & C^2_{(T_v)_j} & C^2_{T_g} \\ C^3_{T_s} & C^3_{q_s} & C^3_{(T_v)_j} & C^3_{T_g} \\ C^4_{T_s} & C^4_{q_s} & C^4_{(T_v)_j} & C^4_{T_g} \end{pmatrix} \times \begin{bmatrix} \Delta T_s \\ \Delta q_s \\ \Delta (T_v)_j \\ \Delta T_g \end{bmatrix} = \begin{bmatrix} F_{T_s} \\ F_{q_s} \\ F_{(T_v)_j} \\ F_{T_g} \end{bmatrix}$$

The matrix coefficients are indexed at top right by the row number (or equation) that they belong to and at bottom right by the column (or prognostic variable) that they correspond to. CLM adjusts the size of matrix *A* in every grid cell according to the actual number of pfts. The matrix size can range from 3x3 for a column with no pfts (e.g., wetland, glacier, bare soil; *npft* equals 1 but *L* + *S* equals 0 in such columns) up to 7x7 for a column with four non-bare ground pfts.

4. Steps Toward Implementation

A fortran routine based on SiB3 subroutine sibslv.F90 was written to fill the coefficients of the matrix of section 3 with realistic data from one time step of a single-point CLM simulation. The main routine calls a matrix solver (subroutine dgesv) and writes the solution as though one CLM time step has passed.

The fortran routine was originally tested in one column with one pft and no snow:

1. The heat capacities of vegetation and canopy air space were set to zero to mimic CLM assumptions. The matrix solution appeared reasonable but values were different from CLM output at the same time step.
2. Finite heat capacities were used for vegetation and canopy air space and the results changed mainly above ground as expected.
3. A $2\Delta t$ smoothing filter was used in Eq. 1c to Eq. 4c following the approach found in SiB3. The results changed mainly above ground because the smoothing was not used below ground.
4. The routine was changed to accommodate multiple pfts. T_s and q_s were made column level variables. The results did not change when setting *npft* = 1.
5. Solving for two or more identical pfts (*npft* > 1) gave same answers for each of the pfts as for the single pft in test #4.
6. Vegetation related variables were set to zero to test the matrix for the case of bare ground. The results changed mainly above ground as expected.
7. The routine was generalized to accommodate snow. The results did not change when *snl* was set to zero.
8. As this document was written, a few errors were found in the definitions of some matrix coefficients, so answers changed. However, the new results look just as reasonable as the old.
9. This new matrix solution will be linked to the CLM as a replacement to the original iterative solution. In CLM the matrix dimensions will be determined dynamically for variable numbers of pfts and snow layers to ensure maximum computational efficiency. Extensive tests will be performed with the new and the

old codes to demonstrate that the new solution works correctly. Some of the tests described earlier in this section will be repeated. Also conservation tests for mass and energy will be performed.

10. We decided to remove the equations solving for soil/snow temperatures other than T_g.
11. Limit canopy and ground evaporation according to the water present on the canopy and in the top soil/snow layer.

5. Necessary Code Changes

List subroutines that were removed, added, or changed. List corresponding sections from Oleson *et al.* (2004) that become obsolete.

Apply the limits recommended by Vidale & Stöckli (2005) (see Eq. B1)?

Change the tridiagonal soil/snow temperature matrix to solve for one less layer.

Talked to Retto (March 15th, 2006):

- He sent the code that includes the water and energy limits. These limits are applied before solving the matrix.
- He offered to review this document. I suggested after we finish reviewing it ourselves.

6. To Do…

Add or just refer to Keith's figures such as 4.1, 5.1, 5.2, 6.1?

Ian (?) suggested that we compile with ATLAS (?)

Bibliography

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Oleson, K. W., Y. Dai, and Coauthors, 2004: Technical description of the Community Land Model (CLM). NCAR Technical Note TN-461+STR, 174.

Vidale, P. L., R. Stöckli, 2005: Prognostic canopy air space solutions for land surface exchanges. *Theor. Appl. Climatol.,* **80**, 245-257.

Add a SiB reference OR Ian's write-up?