The New Prognostic Canopy Air Space Solution in the Community Land Model Version 4 (CLM4)

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Preface

In this document we present an update to the Community Land Model version 3 (CLM3) canopy air space equations from the diagnostic to a prognostic form. We also include an update in the solution to these equations from the iterative to a single-step solution. These changes will appear in CLM4. This document replaces relevant portions of - and is considered an addendum to - the CLM3 technical description of Oleson et al. (2004). For consistency we use the same symbols as Oleson et al. (2004) to represent all variables. This work was supported in part by the X program through grant Y.

Samuel Levis

Boulder, 28 July 2006

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1. Introduction

The Community Land Model (CLM) solves a set of equations once per model time step n for a set of unknowns at time step $n + 1$. The unknowns include leaf temperature, T_v, canopy air space temperature and humidity, T_s and q_s, as well as soil and snow layer temperature, T_i, and moisture (not discussed in this document).

The near-surface state (represented by these T and q variables) responds to an atmospheric forcing (T_{atm}, q_{atm}, radiation, precipitation, wind) from a general circulation model (GCM) or a data set. By solving for the near-surface state, the CLM can calculate heat, water, and radiation fluxes. These fluxes allow for the simulation of two-way land-atmosphere interactions when the CLM is coupled directly to a GCM.

Here we document the model update from an iterative (Oleson et al. 2004) to a single-step solution of the canopy air space variables (Vidale & Stöckli 2005). We also include an update in the corresponding equations from their diagnostic to a prognostic form (Vidale & Stöckli 2005). These updates will appear in CLM version 4.

The new solution of this matrix of prognostic equations simplifies large sections of CLM's code and facilitates the implementation of water isotope tracers in the model. The new solution also reduces sub-daily instability occasionally reported in the heat fluxes simulated by earlier versions of CLM.

2. CLM Heat Flux Equations

To set the stage for the CLM changes documented in subsequent sections, we first summarize the iterative canopy air space solution of prior CLM versions. For a complete derivation, see Oleson et al. (2004) Section 5.

In CLM we define the sensible heat fluxes H, H_g, and H_v (W m^{-2}). These are fluxes from the GCM's reference height ($z_{atm,h} \approx 30$ m above the ground), from the ground, and from the vegetation, each to the height of the canopy air space ($z_{0h} + d$, where z_{0h} (m) is the roughness length for heat and d (m) is the displacement height). By convention a sensible heat flux is positive upwards:

$$
\begin{aligned}
H &= -\frac{\rho_{atm} C_p}{r_{ah}} \left(\theta_{atm} - T_s \right) \\
H_g &= -\frac{\rho_{atm} C_p}{r_{ah'}} \left(T_s - T_g \right) \qquad \text{(Eq. A)} \\
H_v &= -\rho_{atm} C_p \frac{L+S}{r_b} \left(T_s - T_v \right)
\end{aligned}
$$

where ρ_{atm} is the density of atmospheric (moist) air (kg m^{-3}), C_p is the specific heat capacity of dry air (J kg^{-1} K^{-1}), r_{ah} is the aerodynamic resistance to sensible heat transfer (s m^{-1}) between $z_{0h} + d$ and $z_{atm,h}$, r_{ah}' is the aerodynamic resistance to heat transfer (s m^{-1}) between the ground and $z_{0h} + d$, r_b is the leaf boundary layer resistance (s m^{-1}), L and S are the exposed leaf and stem area index values (m^2 leaf or stem area m^{-2} ground), θ_{atm} is the potential temperature (K) at $z_{atm,h}$ (Eq. 5.7 in Oleson et al. (2004)), and T_g is the ground (soil or snow) temperature (K). Variables not defined in this paragraph were defined earlier.

We assume negligible canopy depth, Δz (m), which means negligible heat capacity, $\rho_{atm} C_p \Delta z$, in the canopy air space, so that H_g and H_v must balance H (Eq. 7 in Vidale & Stöckli (2005)):

$$
\left.
\begin{aligned}
\rho_{atm} C_p \Delta z \frac{\Delta T_s}{\Delta t} &= H_g + H_v - H \\
\rho_{atm} C_p \lim_{\Delta z \to 0} (\Delta z) \frac{\Delta T_s}{\Delta t} &= 0
\end{aligned}
\right\} \Rightarrow \quad H = H_g + H_v \qquad \text{(Eq. B)}
$$

where Δt (s) is the model time step.

We solve Eq. B for T_s given θ_{atm}, T_g, L, and S from the previous time step:

$$T_s = \frac{\frac{1}{r_{ah}}\theta_{atm} + \frac{1}{r_{ah}'}T_g + \frac{L+S}{r_b}T_v}{\frac{1}{r_{ah}} + \frac{1}{r_{ah}'} + \frac{L+S}{r_b}} \quad \text{(Eq. C)}$$

In CLM we also define the latent heat fluxes $\lambda_{vap}E$, $\lambda_{vap/sub}E_g$, and $\lambda_{vap}E_v$ (W m^{-2}). These are fluxes from the GCM's reference height for water vapor ($z_{atm,w}$, assumed equal to $z_{atm,h}$), the ground, and the vegetation, each to the height of the canopy air space ($z_{0w} + d$, where z_{0w} is the roughness length for water vapor (m) assumed equal to z_{0h}) . The coefficients λ_{vap} and $\lambda_{vap/sub}$ (J kg^{-1}) convert water vapor flux units (kg m^{-2} s^{-1}) to energy flux units (W m^{-2}) and represent the latent heat of vaporization; $\lambda_{vap/sub}$ represents the latent heat of sublimation when the top layer of soil, or snow if present, contains water in ice form only. By convention a latent heat flux is positive upwards:

$$\begin{aligned}
\lambda_{vap}E &= -\frac{\rho_{atm}\lambda_{vap}}{r_{aw}}\left(q_{atm} - q_s\right) \\
\lambda_{vap/sub}E_g &= -\frac{\rho_{atm}\lambda_{vap/sub}}{r_{aw'}}\left(q_s - q_g\right) \\
\lambda_{vap}E_v &= -\rho_{atm}\lambda_{vap}\left\{f_{wet}\frac{L+S}{r_b} + \frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b + r_s^{sun}} + \frac{L^{sha}}{r_b + r_s^{sha}}\right)\right\}\left(q_s - q_{sat}^{T_v}\right)
\end{aligned} \quad \text{(Eq. D)}$$

where $\lambda_{vap}E_v$ equals the sum of transpiration, $\lambda_{vap}E_v^t$, and canopy evaporation, $\lambda_{vap}E_v^w$:

$$\begin{aligned}
\lambda_{vap}E_v^t &= -\rho_{atm}\lambda_{vap}\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b + r_s^{sun}} + \frac{L^{sha}}{r_b + r_s^{sha}}\right)\left(q_s - q_{sat}^{T_v}\right) \\
\lambda_{vap}E_v^w &= -\rho_{atm}\lambda_{vap}f_{wet}\frac{L+S}{r_b}\left(q_s - q_{sat}^{T_v}\right)
\end{aligned} \quad \text{(Eq. E)}$$

and where f_{wet} is the wetted fraction of the canopy (leaves and stems) and f_{dry} is the fraction of leaves that are dry and able to photosynthesize so that in general $f_{dry} \neq 1 - f_{wet}$ (Eqs. 7.12 and 7.13 in Oleson et al. (2004)); f_{dry} reduces to zero when the soil moisture function that limits transpiration, β_t (Eq. 8.10 in Oleson et al. (2004)), drops to 1 x 10^{-10} or less; f_{dry} equals zero and f_{wet} equals one when dew is present. L^{sun} and L^{sha} are the sunlit and shaded components of L (m^2 m^{-2}), r_s^{sun} and r_s^{sha} are the sunlit and shaded stomatal resistances (s m^{-1}), r_{aw}' and r_{aw} are the aerodynamic resistances to water vapor transfer (s m^{-1}) between the ground and $z_{0w} + d$ and between $z_{0w} + d$ and $z_{atm,w}$, respectively, q_{atm} and q_g are the specific humidities (kg water vapor kg^{-1} air) at $z_{atm,w}$ and the ground, and $q_{sat}^{T_v}$ is the saturated specific humidity (kg kg^{-1}) at temperature T_v.

As in Eq. B for heat capacity, now we neglect the water vapor capacity of the canopy air space (not shown), so that water vapor fluxes E_g and E_v must balance E:

$$E = E_g + E_v \qquad \text{(Eq. F)}$$

We solve Eq. F for q_s given q_{atm}, q_g, f_{wet}, f_{dry}, L, and S from the previous time step:

$$q_s = \frac{\frac{1}{r_{aw}} q_{atm} + \frac{1}{r_{aw}'} q_g + \frac{L+S}{r_b}\left[f_{wet} + \frac{f_{dry} r_b}{L}\left(\frac{L^{sun}}{r_b + r_s^{sun}} + \frac{L^{sha}}{r_b + r_s^{sha}} \right)\right] q_{sat}^{T_v}}{\frac{1}{r_{aw}} + \frac{1}{r_{aw}'} + \frac{L+S}{r_b}\left[f_{wet} + \frac{f_{dry} r_b}{L}\left(\frac{L^{sun}}{r_b + r_s^{sun}} + \frac{L^{sha}}{r_b + r_s^{sha}} \right)\right]} \qquad \text{(Eq. G)}$$

where we show the general form of Eq. G (versus Eqs. 5.90-5.95 in Oleson et al. (2004)).

A canopy energy conservation equation further constrains the solution of T_s, T_v, and q_s. We neglect the heat capacity of vegetation (not shown), so that Eq. H becomes a balance of radiation and heat fluxes:

$$\vec{S}_v - \vec{L}_v - H_v - \lambda_{vap} E_v = 0 \qquad \text{(Eq. H)}$$

Eqs. C, G, and H set the stage for the iterative solution of T_s, T_v, and q_s. The iterative solution is followed by a single-step matrix solution of the soil and snow temperatures, T_i, of the ten soil layers and the zero to five snow layers, where T_i of the top soil/snow layer equals T_g (Figure 1).

3. The Prognostic Equations

Here we update CLM to use the prognostic forms of Eqs. B, F, and H and we add an equation for ground energy conservation to solve for T_s, T_v, q_s, and, now, also T_g. The four energy and mass balance equations are solved as a simultaneous system following the approach of Vidale & Stöckli (2005). All the equations include the change in heat or water vapor storage with time in the left hand side (LHS) versus a sum of heat and radiation fluxes in the right hand side (RHS). Positive or negative signs denote fluxes into or out of the canopy air space (Eqs. 1 and 2), vegetation (Eq. 3), or ground (Eq. 4):

$$\rho_{atm} C_p \Delta z \frac{\Delta T_s}{\Delta t} = H_g + H_v - H \quad \text{(Eq. 1)}$$

$$\rho_{atm} \lambda_{vap} \Delta z \frac{\Delta q_s}{\Delta t} = \lambda_{vap/sub} E_g + \lambda_{vap} E_v - \lambda_{vap} E \quad \text{(Eq. 2)}$$

$$c_v \frac{\Delta T_v}{\Delta t} = \vec{S}_v - \vec{L}_v - H_v - \lambda_{vap} E_v \quad \text{(Eq. 3)}$$

$$c_{soil,1+snl} \frac{\Delta T_g}{\Delta t} = \vec{S}_g - \vec{L}_g - H_g - \lambda_{vap/sub} E_g - F_{1+snl} \quad \text{(Eq. 4)}$$

where c_v (J m^{-2} K^{-1}) is the heat capacity of vegetation equal to $(L+S)C_{liq}W_{l+s} + C_{liq}W_{can}$, where C_{liq} is the specific heat capacity of water (J kg^{-1} K^{-1}), W_{l+s} is the amount of water in leaves and stems set to 0.2 kg m^{-2} leaf plus stem area, and W_{can} is water on the canopy surface per unit ground area (kg m^{-2}). Similarly , $c_{soil,1+snl}$ (J m^{-2} K^{-1}) is the heat capacity of the top soil/snow layer (index 1+*snl*) where *snl* is the number of snow layers ranging from 0 to –5. The top layer index is 1 with no snow and -4 with five snow layers. Oleson et al. (2004) write $c_{soil,1+snl}$ as the product of the layer's volumetric heat capacity, c_{1+snl}, by the layer's thickness, Δz_{1+snl}. $\vec{S}_v$, $\vec{L}_v$, $\vec{S}_g$, and $\vec{L}_g$ are the vegetation and ground's net solar and net longwave radiation fluxes (W m^{-2}), respectively. F_{1+snl} is the ground heat flux (W m^{-2}) and, therefore, the lower boundary condition for Eqs. 1 to 4.

Eqs. 1 to 4 can be solved simultaneously using a matrix solver, where the matrix takes the general form:

$$A \cdot x = B$$

(Eq. 5)

$$\begin{pmatrix} C_{T_s}^1 & C_{q_s}^1 & C_{T_v}^1 & C_{T_g}^1 \\ C_{T_s}^2 & C_{q_s}^2 & C_{T_v}^2 & C_{T_g}^2 \\ C_{T_s}^3 & C_{q_s}^3 & C_{T_v}^3 & C_{T_g}^3 \\ C_{T_s}^4 & C_{q_s}^4 & C_{T_v}^4 & C_{T_g}^4 \end{pmatrix} \cdot \begin{bmatrix} \Delta T_s \\ \Delta q_s \\ \Delta T_v \\ \Delta T_g \end{bmatrix} = \begin{bmatrix} F_{T_s} \\ F_{q_s} \\ F_{T_v} \\ F_{T_g} \end{bmatrix}$$

However, before entering the terms from Eqs. 1 to 4 into Eq. 5, we rewrite the equations so as to employ a time-differencing scheme that improves the accuracy of the solution.

4. Numerical Implementation

We transform Eqs. 1 to 4 algebraically assuming an implicit scheme with explicit coefficients. Terms that depend on the variables that we are solving for are indexed $n + 1$ (implicit scheme), while other terms, e.g. solar radiation and various coefficients, are indexed n (explicit coefficients) (Kalnay and Kanamitsu 1988):

$$\rho_{atm} C_p \Delta z \frac{\Delta T_s}{\Delta t} = H_g^{n+1} + H_v^{n+1} - H^{n+1} \qquad \text{(Eq. 1.0)}$$

$$\rho_{atm}\lambda_{vap}\Delta z\frac{\Delta q_s}{\Delta t}=\lambda_{vap/sub}E_g^{n+1}+\lambda_{vap}E_v^{n+1}-\lambda_{vap}E^{n+1} \qquad \text{(Eq. 2.0)}$$

$$c_v\frac{\Delta T_v}{\Delta t}=\vec{S}_v^n-\vec{L}_v^{n+1}-H_v^{n+1}-\lambda_{vap}E_v^{n+1} \qquad \text{(Eq. 3.0)}$$

$$c_{soil,1+snl}\frac{\Delta T_g}{\Delta t}=\vec{S}_g^n-\vec{L}_g^{n+1}-H_g^{n+1}-\lambda_{vap/sub}E_g^{n+1}-F_{1+snl}^{n+1} \qquad \text{(Eq. 4.0)}$$

The algebraic transformations necessary to solve Eqs. 1 to 4 include:

1. Carrying the $n + 1$ terms to the LHS.
2. Adding the corresponding n terms to both sides of the equation.
3. Expanding all terms (following Eqs. A, D, and E).
4. Rearranging the LHS by variable rather than time step.

Let us apply these steps to Eq. 1.0:

$$\rho_{atm}C_p\Delta z\frac{\Delta T_s}{\Delta t}+H^{n+1}-H_g^{n+1}-H_v^{n+1}=0 \qquad \text{(Eq. 1.1)}$$

$$\rho_{atm}C_p\Delta z\frac{\Delta T_s}{\Delta t}+H^{n+1}-H^n-H_g^{n+1}+H_g^n-H_v^{n+1}+H_v^n=H_v^n+H_g^n-H^n \qquad \text{(Eq. 1.2)}$$

$$\begin{aligned}
&\frac{\rho_{atm}C_p\Delta z}{\Delta t}\left(T_s^{n+1}-T_s^n\right)-\frac{\rho_{atm}C_p}{r_{ah}}\left(\theta_{atm}^n-T_s^{n+1}\right)+\frac{\rho_{atm}C_p}{r_{ah}}\left(\theta_{atm}^n-T_s^n\right)\\
&+\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^{n+1}-T_g^{n+1}\right)-\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^n-T_g^n\right)\\
&+\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^{n+1}-T_v^{n+1}\right)-\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^n-T_v^n\right)\\
&=-\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^n-T_v^n\right)-\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^n-T_g^n\right)+\frac{\rho_{atm}C_p}{r_{ah}}\left(\theta_{atm}^n-T_s^n\right)
\end{aligned} \qquad \text{(Eq. 1.3)}$$

$$\left(\frac{\rho_{atm}C_p\Delta z}{\Delta t}+\frac{\rho_{atm}C_p}{r_{ah}}+\rho_{atm}C_p\frac{L+S}{r_b}+\frac{\rho_{atm}C_p}{r_{ah'}}\right)\left(T_s^{n+1}-T_s^n\right)$$
$$-\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_g^{n+1}-T_g^n\right)-\rho_{atm}C_p\frac{L+S}{r_b}\left(T_v^{n+1}-T_v^n\right) \qquad \text{(Eq. 1.4)}$$
$$=-\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^n-T_v^n\right)-\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^n-T_g^n\right)+\frac{\rho_{atm}C_p}{r_{ah}}\left(\theta_{atm}^n-T_s^n\right)$$

In Eq. 1.4 the θ_{atm} term drops out because we write both H^n and H^{n+1} in terms of θ_{atm}^n, which is a function of T_{atm}^n. T_{atm}^{n+1} will not be available until the next time step from a data set or a GCM in offline and coupled mode, respectively.

Now we apply the same transformations to Eq. 2.0:

$$\rho_{atm}\lambda_{vap}\Delta z\frac{\Delta q_s}{\Delta t}+\lambda_{vap}E^{n+1}-\lambda_{vap/sub}E_g^{n+1}-\lambda_{vap}E_v^{n+1}=0 \qquad \text{(Eq. 2.1)}$$

$$\rho_{atm}\lambda_{vap}\Delta z\frac{\Delta q_s}{\Delta t}+\lambda_{vap}E^{n+1}-\lambda_{vap}E^n-\lambda_{vap/sub}E_g^{n+1}+\lambda_{vap/sub}E_g^n-\lambda_{vap}E_v^{n+1}+\lambda_{vap}E_v^n$$
$$=\lambda_{vap}E_v^n+\lambda_{vap/sub}E_g^n-\lambda_{vap}E^n \qquad \text{(Eq. 2.2)}$$

$$\frac{\rho_{atm}\lambda_{vap}\Delta z}{\Delta t}\left(q_s^{n+1}-q_s^n\right)-\frac{\rho_{atm}\lambda_{vap}}{r_{aw}}\left(q_{atm}^n-q_s^{n+1}\right)+\frac{\rho_{atm}\lambda_{vap}}{r_{aw}}\left(q_{atm}^n-q_s^n\right)$$
$$+\frac{\rho_{atm}\lambda_{vap/sub}}{r_{aw'}}\left(q_s^{n+1}-q_g^{n+1}\right)-\frac{\rho_{atm}\lambda_{vap/sub}}{r_{aw'}}\left(q_s^n-q_g^n\right)$$
$$+\rho_{atm}\lambda_{vap}\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(q_s^{n+1}-q_{sat}^{T_v^{n+1}}\right)$$
$$-\rho_{atm}\lambda_{vap}\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(q_s^n-q_{sat}^{T_v^n}\right) \qquad \text{(Eq. 2.3)}$$
$$=-\rho_{atm}\lambda_{vap}\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(q_s^n-q_{sat}^{T_v^n}\right)$$
$$-\frac{\rho_{atm}\lambda_{vap/sub}}{r_{aw'}}\left(q_s^n-q_g^n\right)+\frac{\rho_{atm}\lambda_{vap}}{r_{aw}}\left(q_{atm}^n-q_s^n\right)$$

$$
\rho_{atm}\left\{\begin{array}{l}\dfrac{\lambda_{vap}\Delta z}{\Delta t}+\dfrac{\lambda_{vap}}{r_{aw}}+\dfrac{\lambda_{vap/sub}}{r_{aw'}} \\ +\lambda_{vap} f_{wet}\dfrac{L+S}{r_b}+\dfrac{\lambda_{vap} f_{dry}}{L}\left(\dfrac{L^{sun}}{r_b+r_s^{sun}}+\dfrac{L^{sha}}{r_b+r_s^{sha}}\right)\end{array}\right\}\left(q_s^{n+1}-q_s^n\right)
$$

$$
-\rho_{atm}\lambda_{vap}\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(q_{sat}^{T_v^{n+1}}-q_{sat}^{T_v^n}\right)
$$

$$
-\frac{\rho_{atm}\lambda_{vap/sub}}{r_{aw'}}\left(q_g^{n+1}-q_g^n\right) \qquad \text{(Eq. 2.4)}
$$

$$
=-\rho_{atm}\lambda_{vap}\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(q_s^n-q_{sat}^{T_v^n}\right)
$$

$$
-\frac{\rho_{atm}\lambda_{vap/sub}}{r_{aw'}}\left(q_s^n-q_g^n\right)+\frac{\rho_{atm}\lambda_{vap}}{r_{aw}}\left(q_{atm}^n-q_s^n\right)
$$

In Eq. 2.4 the q_{atm} term drops out because we write both $\lambda_{vap}E^n$ and $\lambda_{vap}E^{n+1}$ in terms of q_{atm}^n. The n + 1 value of q will not become available until the next time step from a data set or a GCM.

Next we work with Eq. 3.0:

$$
c_v\frac{\Delta T_v}{\Delta t}+H_v^{n+1}+\lambda_{vap}E_v^{n+1}+\vec{L}_v^{n+1}=\vec{S}_v^n \qquad \text{(Eq. 3.1)}
$$

$$
c_v\frac{\Delta T_v}{\Delta t}+H_v^{n+1}-H_v^n+\lambda_{vap}E_v^{n+1}-\lambda_{vap}E_v^n+\vec{L}_v^{n+1}-\vec{L}_v^n=\vec{S}_v^n-\vec{L}_v^n-H_v^n-\lambda_{vap}E_v^n \qquad \text{(Eq. 3.2)}
$$

$$
\begin{aligned}
&\frac{c_v}{\Delta t}\left(T_v^{n+1}-T_v^n\right)-\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^{n+1}-T_v^{n+1}\right)+\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^n-T_v^n\right)\\
&-\rho_{atm}\lambda_{vap}\left[f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_s^{n+1}-q_{sat}^{T_v^{n+1}}\right)\\
&+\rho_{atm}\lambda_{vap}\left[f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_s^n-q_{sat}^{T_v^n}\right)\\
&+\left.\frac{d\vec{L}_v}{dT_g}\right|_n\left(T_g^{n+1}-T_g^n\right)+\left.\frac{d\vec{L}_v}{dT_v}\right|_n\left(T_v^{n+1}-T_v^n\right)\\
&=\vec{S}_v^n-\vec{L}_v^n+\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^n-T_v^n\right)\\
&+\rho_{atm}\lambda_{vap}\left[f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_s^n-q_{sat}^{T_v^n}\right)
\end{aligned}
\qquad \text{(Eq. 3.3)}
$$

$$
\begin{aligned}
&\left[\frac{c_v}{\Delta t}+4\left[2-\varepsilon_v\left(1-\varepsilon_g\right)\right]\varepsilon_v\sigma\left(T_v^n\right)^3+\rho_{atm}C_p\frac{L+S}{r_b}\right]\left(T_v^{n+1}-T_v^n\right)\\
&-\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^{n+1}-T_s^n\right)\\
&+\rho_{atm}\lambda_{vap}\left[f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_{sat}^{T_v^{n+1}}-q_{sat}^{T_v^n}\right)\\
&-\rho_{atm}\lambda_{vap}\left[f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_s^{n+1}-q_s^n\right)\\
&-4\varepsilon_v\varepsilon_g\sigma\left(T_g^n\right)^3\left(T_g^{n+1}-T_g^n\right)\\
&=\vec{S}_v^n-\vec{L}_v^n+\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^n-T_v^n\right)\\
&+\rho_{atm}\lambda_{vap}\left[f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_s^n-q_{sat}^{T_v^n}\right)
\end{aligned}
\qquad \text{(Eq. 3.4)}
$$

where in Eq. 3.3 we replace $\vec{L}_v^{n+1}-\vec{L}_v^n$ with $\left.\frac{d\vec{L}_v}{dT_g}\right|_n\left(T_g^{n+1}-T_g^n\right)+\left.\frac{d\vec{L}_v}{dT_v}\right|_n\left(T_v^{n+1}-T_v^n\right)$ and in Eq. 3.4 with $-4\varepsilon_v\varepsilon_g\sigma\left(T_g^n\right)^3\left(T_g^{n+1}-T_g^n\right)+4\left[2-\varepsilon_v\left(1-\varepsilon_g\right)\right]\varepsilon_v\sigma\left(T_v^n\right)^3\left(T_v^{n+1}-T_v^n\right)$, where ε_v and ε_g

are the vegetation and ground emissivities (fractions) and σ is the Stefan-Boltzmann constant (W m^{-2} K^{-4}). The derivatives were calculated on Eq. 4.19 of Oleson et al. (2004).

Lastly we apply steps 1 to 4 to Eq. 4.0:

$$c_{soil,1+snl}\frac{\Delta T_g}{\Delta t}+H_g^{n+1}+\lambda_{vap/sub}E_g^{n+1}+F_{1+snl}^{n+1}+\vec{L}_g^{n+1}=\vec{S}_g^{n} \qquad \text{(Eq. 4.1)}$$

$$\begin{aligned}&c_{soil,1+snl}\frac{\Delta T_g}{\Delta t}+H_g^{n+1}-H_g^{n}+\lambda_{vap/sub}E_g^{n+1}-\lambda_{vap/sub}E_g^{n}+F_{1+snl}^{n+1}-F_{1+snl}^{n}+\vec{L}_g^{n+1}-\vec{L}_g^{n}\\&=\vec{S}_g^{n}-\vec{L}_g^{n}-H_g^{n}-\lambda_{vap/sub}E_g^{n}-F_{1+snl}^{n}\end{aligned} \qquad \text{(Eq. 4.2)}$$

$$\begin{aligned}&\frac{c_{soil,1+snl}}{\Delta t}\left(T_g^{n+1}-T_g^{n}\right)-\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^{n+1}-T_g^{n+1}\right)+\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^{n}-T_g^{n}\right)\\&-\frac{\rho_{atm}\lambda_{vap/sub}}{r_{aw'}}\left(q_s^{n+1}-q_g^{n+1}\right)+\frac{\rho_{atm}\lambda_{vap/sub}}{r_{aw'}}\left(q_s^{n}-q_g^{n}\right)\\&-\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\left(T_g^{n+1}-T_{2+snl}^{n}\right)+\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\left(T_g^{n}-T_{2+snl}^{n}\right)\\&+\left.\frac{d\vec{L}_g}{dT_g}\right|_n\left(T_g^{n+1}-T_g^{n}\right)+\left.\frac{d\vec{L}_g}{dT_v}\right|_n\left(T_v^{n+1}-T_v^{n}\right)\\&=\vec{S}_g^{n}-\vec{L}_g^{n}+\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^{n}-T_g^{n}\right)+\frac{\rho_{atm}\lambda_{vap/sub}}{r_{aw'}}\left(q_s^{n}-q_g^{n}\right)+\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\left(T_g^{n}-T_{2+snl}^{n}\right)\end{aligned} \qquad \text{(Eq. 4.3)}$$

$$\begin{aligned}&\left[\frac{c_{soil,1+snl}}{\Delta t}+\frac{\rho_{atm}C_p}{r_{ah'}}+4\varepsilon_g\sigma\left(T_g^{n}\right)^3-\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\right]\left(T_g^{n+1}-T_g^{n}\right)\\&-\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^{n+1}-T_s^{n}\right)-\frac{\rho_{atm}\lambda_{vap/sub}}{r_{aw'}}\left(q_s^{n+1}-q_s^{n}\right)+\frac{\rho_{atm}\lambda_{vap/sub}}{r_{aw'}}\left(q_g^{n+1}-q_g^{n}\right)\\&-4\varepsilon_v\varepsilon_g\sigma\left(T_v^{n}\right)^3\left(T_v^{n+1}-T_v^{n}\right)\\&=\vec{S}_g^{n}-\vec{L}_g^{n}+\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^{n}-T_g^{n}\right)+\frac{\rho_{atm}\lambda_{vap/sub}}{r_{aw'}}\left(q_s^{n}-q_g^{n}\right)+\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\left(T_g^{n}-T_{2+snl}^{n}\right)\end{aligned} \qquad \text{(Eq. 4.4)}$$

where in Eq. 4.3 we replace $\vec{L}_g^{n+1} - \vec{L}_g^n$ with $\left.\frac{d\vec{L}_g}{dT_g}\right|_n \left(T_g^{n+1} - T_g^n\right) + \left.\frac{d\vec{L}_g}{dT_v}\right|_n \left(T_v^{n+1} - T_v^n\right)$ and in Eq. 4.4 with $4\varepsilon_g \sigma \left(T_g^n\right)^3 \left(T_g^{n+1} - T_g^n\right) - 4\varepsilon_v \varepsilon_g \sigma \left(T_v^n\right)^3 \left(T_v^{n+1} - T_v^n\right)$. Here $\lambda[z_{h,1+snl}]$ (W m^{-1} K^{-1}), not to be confused with $\lambda_{vap/sub}$, is the thermal conductivity at the interface between the top and second soil/snow layers, and $z_{h,1+snl}$ (m) refers to the depth of that interface, while z_{1+snl} and z_{2+snl} (m) are the depths of the top and second soil/snow layers. In Eq. 4.4 the T_{2+snl} term drops out because we write both F_{1+snl}^n and F_{1+snl}^{n+1} in terms of T_{2+snl}^n. T_{2+snl}^{n+1} becomes available later in the time step when the matrix of soil/snow temperature equations is solved and where the flux F_{1+snl}^{n+1} becomes the upper boundary condition. These two variables are equivalent to T_g^{n+1} and G^{n+1} in prior CLM versions (Figure 1 vs. Figure 2).

Eqs. 2.4 to 4.4 appear to include the additional unknowns $q_{sat}^{T_v}$ and q_g. However, if we assume that $\frac{dq_{sat}^T}{dT} = \frac{q_{sat}^{T^{n+1}} - q_{sat}^{T^n}}{T^{n+1} - T^n}$, where q_{sat}^T is the saturated specific humidity at temperature *T*, then we can substitute $q_{sat}^{T_v^{n+1}} - q_{sat}^{T_v^n}$ with $\frac{dq_{sat}^{T_v}}{dT_v}\left(T_v^{n+1} - T_v^n\right)$. If we also assume $\frac{dq_g}{dT_g} = \alpha \frac{dq_{sat}^{T_g}}{dT_g}$ because $q_g = \alpha q_{sat}^{T_g}$ (specific humidity at the ground as a function of saturated specific humidity at the ground; Section 5.2 of Oleson et al. (2004)), then we can substitute $q_g^{n+1} - q_g^n$ with $\frac{dq_g}{dT_g}\left(T_g^{n+1} - T_g^n\right)$:

$$\rho_{atm}\left\{\begin{array}{l}\dfrac{\lambda_{vap}\Delta z}{\Delta t}+\dfrac{\lambda_{vap}}{r_{aw}}+\dfrac{\lambda_{vap/sub}}{r_{aw'}}\\ +\lambda_{vap}f_{wet}\dfrac{L+S}{r_b}+\dfrac{\lambda_{vap}f_{dry}}{L}\left(\dfrac{L^{sun}}{r_b+r_s^{sun}}+\dfrac{L^{sha}}{r_b+r_s^{sha}}\right)\end{array}\right\}\left(q_s^{n+1}-q_s^n\right)$$

$$-\rho_{atm}\lambda_{vap}\frac{dq_{sat}^{T_v}}{dT_v}\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(T_v^{n+1}-T_v^n\right)$$

$$-\frac{\rho_{atm}\lambda_{vap/sub}}{r_{aw'}}\frac{dq_g}{dT_g}\left(T_g^{n+1}-T_g^n\right) \qquad \text{(Eq. 2.5)}$$

$$=-\rho_{atm}\lambda_{vap}\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\left(q_s^n-q_{sat}^{T_v^n}\right)$$

$$-\frac{\rho_{atm}\lambda_{vap/sub}}{r_{aw'}}\left(q_s^n-q_g^n\right)+\frac{\rho_{atm}\lambda_{vap}}{r_{aw}}\left(q_{atm}^n-q_s^n\right)$$

$$\left[\frac{c_v}{\Delta t}+4\left[2-\varepsilon_v\left(1-\varepsilon_g\right)\right]\varepsilon_v\sigma\left(T_v^n\right)^3+\rho_{atm}C_p\frac{L+S}{r_b}\right.$$

$$\left.+\rho_{atm}\lambda_{vap}\frac{dq_{sat}^{T_v}}{dT_v}\left\{f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right\}\right]\left(T_v^{n+1}-T_v^n\right)$$

$$-\rho_{atm}\lambda_{vap}\left[f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_s^{n+1}-q_s^n\right) \qquad \text{(Eq. 3.5)}$$

$$-\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^{n+1}-T_s^n\right)-4\varepsilon_v\varepsilon_g\sigma\left(T_g^n\right)^3\left(T_g^{n+1}-T_g^n\right)$$

$$=\vec{S}_v^n-\vec{L}_v^n+\rho_{atm}C_p\frac{L+S}{r_b}\left(T_s^n-T_v^n\right)$$

$$+\rho_{atm}\lambda_{vap}\left[f_{wet}\frac{L+S}{r_b}+\frac{f_{dry}}{L}\left(\frac{L^{sun}}{r_b+r_s^{sun}}+\frac{L^{sha}}{r_b+r_s^{sha}}\right)\right]\left(q_s^n-q_{sat}^{T_v^n}\right)$$

$$\left[\frac{c_{soil,1+snl}}{\Delta t}+\frac{\rho_{atm}C_p}{r_{ah'}}+\frac{\rho_{atm}\lambda_{vap/sub}}{r_{aw'}}\frac{dq_g}{dT_g}+4\varepsilon_g\sigma\left(T_g^n\right)^3-\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\right]\left(T_g^{n+1}-T_g^n\right)$$

$$-4\varepsilon_v\varepsilon_g\sigma\left(T_v^n\right)^3\left(T_v^{n+1}-T_v^n\right)-\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^{n+1}-T_s^n\right)-\frac{\rho_{atm}\lambda_{vap/sub}}{r_{aw'}}\left(q_s^{n+1}-q_s^n\right) \qquad \text{(Eq. 4.5)}$$

$$=\vec{S}_g^n-\vec{L}_g^n+\frac{\rho_{atm}C_p}{r_{ah'}}\left(T_s^n-T_g^n\right)+\frac{\rho_{atm}\lambda_{vap/sub}}{r_{aw'}}\left(q_s^n-q_g^n\right)+\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\left(T_g^n-T_{2+snl}^n\right)$$

The terms from Eq. 1.5 (not shown; same as Eq. 1.4) and Eqs. 2.5, 3.5, and 4.5 could enter the matrix equation shown in Eq. 5. However, first we must tackle certain issues regarding the implementation of these equations in CLM.

5. CLM Implementation

Our system of equations includes column level variables, e.g. T_s, q_s, and T_g, and plant functional type (pft) level variables, e.g. T_v and the resistance terms. Assuming multiple pfts per column, we generalize Eqs. 1.5 to 4.5 to a form applicable to the CLM:

$$
\begin{aligned}
&\sum_{j=1}^{npft}\left[(wt)_j\left(\frac{\Delta z_j}{\Delta t}+\frac{1}{(r_{ah})_j}+\frac{L_j+S_j}{(r_b)_j}+\frac{1}{(r_{ah'})_j}\right)\rho_{atm}C_p\right]\left(T_s^{n+1}-T_s^n\right)\\
&-\sum_{j=1}^{npft}\left[(wt)_j\frac{\rho_{atm}C_p}{(r_{ah'})_j}\right]\left(T_g^{n+1}-T_g^n\right)\\
&-\sum_{j=1}^{npft}\left[(wt)_j\frac{L_j+S_j}{(r_b)_j}\rho_{atm}C_p\left[(T_v^{n+1})_j-(T_v^n)_j\right]\right]\\
&=\sum_{j=1}^{npft}\left[(wt)_j\left(\begin{array}{l}-\rho_{atm}C_p\dfrac{L_j+S_j}{(r_b)_j}\left(T_s^n-(T_v^n)_j\right)\\ -\dfrac{\rho_{atm}C_p}{(r_{ah'})_j}\left(T_s^n-T_g^n\right)+\dfrac{\rho_{atm}C_p}{(r_{ah})_j}\left(\theta_{atm}^n-T_s^n\right)\end{array}\right)\right]
\end{aligned}
\qquad \text{(Eq. 1.6)}
$$

$$
\begin{aligned}
&\sum_{j=1}^{npft}\left[(wt)_j\,\rho_{atm}\left\{\begin{aligned}&\frac{\lambda_{vap}\Delta z_j}{\Delta t}+\frac{\lambda_{vap}}{(r_{aw})_j}+\frac{\lambda_{vap/sub}}{(r_{aw'})_j}+\lambda_{vap}(f_{wet})_j\,\frac{L_j+S_j}{(r_b)_j}\\&+\frac{\lambda_{vap}(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j}+\frac{L_j^{sha}}{(r_b)_j+(r_s^{sha})_j}\right)\end{aligned}\right\}\right]\left(q_s^{n+1}-q_s^n\right)\\
&-\sum_{j=1}^{npft}\left[(wt)_j\,\rho_{atm}\lambda_{vap}\,\frac{dq_{sat}^{(T_v)_j}}{d(T_v)_j}\left\{\begin{aligned}&(f_{wet})_j\,\frac{L_j+S_j}{(r_b)_j}\\&+\frac{(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j}+\frac{L_j^{sha}}{(r_b)_j+(r_s^{sha})_j}\right)\end{aligned}\right\}\left[(T_v^{n+1})_j-(T_v^n)_j\right]\right]\\
&-\sum_{j=1}^{npft}\left[(wt)_j\,\frac{\rho_{atm}\lambda_{vap/sub}}{(r_{aw'})_j}\frac{dq_g}{dT_g}\right]\left(T_g^{n+1}-T_g^n\right)\\
&=\sum_{j=1}^{npft}\left[(wt)_j\left(\begin{aligned}&-\rho_{atm}\lambda_{vap}\left\{\begin{aligned}&(f_{wet})_j\,\frac{L_j+S_j}{(r_b)_j}\\&+\frac{(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j}+\frac{L_j^{sha}}{(r_b)_j+(r_s^{sha})_j}\right)\end{aligned}\right\}\left(q_s^n-q_{sat}^{(T_v^n)_j}\right)\\&-\frac{\rho_{atm}\lambda_{vap/sub}}{(r_{aw'})_j}\left(q_s^n-q_g^n\right)+\frac{\rho_{atm}\lambda_{vap}}{(r_{aw})_j}\left(q_{atm}^n-q_s^n\right)\end{aligned}\right)\right]
\end{aligned}
\qquad \text{(Eq. 2.6)}
$$

$$
\begin{aligned}
&\left[\frac{(c_v)_j}{\Delta t}+4\left[2-(\varepsilon_v)_j\left(1-\varepsilon_g\right)\right](\varepsilon_v)_j\,\sigma\left(T_v^n\right)_j^3+\rho_{atm}C_p\,\frac{L_j+S_j}{(r_b)_j}\right.\\
&\left.+\rho_{atm}\lambda_{vap}\,\frac{dq_{sat}^{(T_v)_j}}{d(T_v)_j}\left\{\begin{aligned}&(f_{wet})_j\,\frac{L_j+S_j}{(r_b)_j}\\&+\frac{(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j}+\frac{L_j^{sha}}{(r_b)_j+(r_s^{sha})_j}\right)\end{aligned}\right\}\right]\left[(T_v^{n+1})_j-(T_v^n)_j\right]\\
&-\rho_{atm}\lambda_{vap}\left[(f_{wet})_j\,\frac{L_j+S_j}{(r_b)_j}+\frac{(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j}+\frac{L_j^{sha}}{(r_b)_j+(r_s^{sha})_j}\right)\right]\left(q_s^{n+1}-q_s^n\right)\\
&-\rho_{atm}C_p\,\frac{L_j+S_j}{(r_b)_j}\left(T_s^{n+1}-T_s^n\right)-4(\varepsilon_v)_j\,\varepsilon_g\,\sigma\left(T_g^n\right)^3\left(T_g^{n+1}-T_g^n\right)\\
&=(\vec{S}_v^n)_j-(\vec{L}_v^n)_j+\rho_{atm}C_p\,\frac{L_j+S_j}{(r_b)_j}\left(T_s^n-(T_v^n)_j\right)\\
&+\rho_{atm}\lambda_{vap}\left[(f_{wet})_j\,\frac{L_j+S_j}{(r_b)_j}+\frac{(f_{dry})_j}{L_j}\left(\frac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j}+\frac{L_j^{sha}}{(r_b)_j+(r_s^{sha})_j}\right)\right]\left(q_s^n-q_{sat}^{(T_v^n)_j}\right)
\end{aligned}
\qquad \text{(Eq. 3.6)}
$$

$$
\begin{aligned}
&\sum_{j=1}^{npft}\left[(wt)_j\left\{\frac{c_{soil,1+snl}}{\Delta t}+\frac{\rho_{atm}C_p}{(r_{ah'})_j}+\frac{\rho_{atm}\lambda_{vap/sub}}{(r_{aw'})_j}\frac{dq_g}{dT_g}+4\varepsilon_g\sigma\left(T_g^n\right)^3-\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\right\}\right]\left(T_g^{n+1}-T_g^n\right)\\
&-\sum_{j=1}^{npft}\left[(wt)_j 4\varepsilon_v\varepsilon_g\sigma\left(T_v^n\right)_j^3\left[(T_v^{n+1})_j-(T_v^n)_j\right]\right]\\
&-\sum_{j=1}^{npft}\left[(wt)_j\frac{\rho_{atm}C_p}{(r_{ah'})_j}\right]\left(T_s^{n+1}-T_s^n\right)-\sum_{j=1}^{npft}\left[(wt)_j\frac{\rho_{atm}\lambda_{vap/sub}}{(r_{aw'})_j}\right]\left(q_s^{n+1}-q_s^n\right) \qquad \text{(Eq. 4.6)}\\
&=\sum_{j=1}^{npft}\left[(wt)_j\left(\begin{aligned}&\vec{S}_g^n-\vec{L}_g^n+\frac{\rho_{atm}C_p}{(r_{ah'})_j}\left(T_s^n-T_g^n\right)+\frac{\rho_{atm}\lambda_{vap/sub}}{(r_{aw'})_j}\left(q_s^n-q_g^n\right)\\&+\frac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl}-z_{1+snl}}\left(T_g^n-T_{2+snl}^n\right)\end{aligned}\right)\right]
\end{aligned}
$$

where j is the pft index ranging from 1 to *npft* (the number of pfts present in a soil column) and $(wt)_j$ is the fraction of the column occupied by pft j, where $\sum_{j=1}^{npft}(wt)_j=1$. CLM includes bare ground in the same soil column as the vegetation and gives it a pft index. The fraction of the column with bare ground has $L_j=0$ and $S_j=0$. The matrix of equations is solved once per CLM soil column and Eqs. 1.6, 2.6, and 4.6 each appear once in the matrix. However, Eq. 3.6 appears once for each existing pft except bare ground. For bare ground all terms reduce to zero in Eq. 3.6 (Figure 2).

In matrix coefficient form, C_x^y is the matrix coefficient in row y that is multiplied by $\Delta x=x^{n+1}-x^n$, where x^{n+1} is an unknown variable such as T_s^{n+1}. Each row y corresponds to an equation such as Eq. 1.6 above, where F_x is the RHS term of that equation. We write Eqs. 1.6 to 4.6 in matrix coefficient form:

$$C_{T_s}^1 = \sum_{j=1}^{npft}\left[(wt)_j\left(\frac{\Delta z_j}{\Delta t}+\frac{1}{(r_{ah})_j}+\frac{L_j+S_j}{(r_b)_j}+\frac{1}{(r_{ah'})_j}\right)\rho_{atm}C_p\right]$$

$$C_{(T_v)_j}^1 = -(wt)_j\frac{L_j+S_j}{(r_b)_j}\rho_{atm}C_p \quad \text{for } j=1,2,\ldots,npft$$

$$C_{T_g}^1 = -\sum_{j=1}^{npft}\left[(wt)_j\frac{\rho_{atm}C_p}{(r_{ah'})_j}\right] \qquad \text{(Eq. 1.7)}$$

$$F_{T_s} = \sum_{j=1}^{npft}\left[(wt)_j\left(\begin{array}{l} -\rho_{atm}C_p\dfrac{L_j+S_j}{(r_b)_j}\left(T_s^n-(T_v^n)_j\right) \\ -\dfrac{\rho_{atm}C_p}{(r_{ah'})_j}\left(T_s^n-T_g^n\right)+\dfrac{\rho_{atm}C_p}{(r_{ah})_j}\left(\overline{\theta}_{atm}^n-T_s^n\right)\end{array}\right)\right]$$

$$C_{q_s}^2 = \sum_{j=1}^{npft}\left[(wt)_j\rho_{atm}\left\{\begin{array}{l}\dfrac{\lambda_{vap}\Delta z_j}{\Delta t}+\dfrac{\lambda_{vap}}{(r_{aw})_j}+\dfrac{\lambda_{vap/sub}}{(r_{aw'})_j}+\lambda_{vap}(f_{wet})_j\dfrac{L_j+S_j}{(r_b)_j} \\ +\dfrac{\lambda_{vap}(f_{dry})_j}{L_j}\left(\dfrac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j}+\dfrac{L_j^{sha}}{(r_b)_j+(r_s^{sha})_j}\right)\end{array}\right\}\right]$$

$$C_{(T_v)_j}^2 = -(wt)_j\rho_{atm}\lambda_{vap}\frac{dq_{sat}^{(T_v)_j}}{d(T_v)_j}\left\{\begin{array}{l}(f_{wet})_j\dfrac{L_j+S_j}{(r_b)_j} \\ +\dfrac{(f_{dry})_j}{L_j}\left(\dfrac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j}+\dfrac{L_j^{sha}}{(r_b)_j+(r_s^{sha})_j}\right)\end{array}\right\}$$

for $j=1,2,\ldots,npft$

$$C_{T_g}^2 = -\sum_{j=1}^{npft}\left[(wt)_j\frac{\rho_{atm}\lambda_{vap/sub}}{(r_{aw'})_j}\frac{dq_g}{dT_g}\right]$$

$$F_{q_s} = \sum_{j=1}^{npft}\left[(wt)_j\left(\begin{array}{l}-\rho_{atm}\lambda_{vap}\left\{\begin{array}{l}(f_{wet})_j\dfrac{L_j+S_j}{(r_b)_j} \\ +\dfrac{(f_{dry})_j}{L_j}\left(\dfrac{L_j^{sun}}{(r_b)_j+(r_s^{sun})_j}+\dfrac{L_j^{sha}}{(r_b)_j+(r_s^{sha})_j}\right)\end{array}\right\}\left(q_s^n-q_{sat}^{(T_v^n)_j}\right) \\ -\dfrac{\rho_{atm}\lambda_{vap/sub}}{(r_{aw'})_j}\left(q_s^n-q_g^n\right)+\dfrac{\rho_{atm}\lambda_{vap}}{(r_{aw})_j}\left(q_{atm}^n-q_s^n\right)\end{array}\right)\right] \qquad \text{(Eq. 2.7)}$$

$$C_{T_s}^{2+j} = -\rho_{atm} C_p \frac{L_j + S_j}{(r_b)_j}$$

$$C_{q_s}^{2+j} = -\rho_{atm} \lambda_{vap} \left\{ (f_{wet})_j \frac{L_j + S_j}{(r_b)_j} + \frac{(f_{dry})_j}{L_j} \left(\frac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j + (r_s^{sha})_j} \right) \right\}$$

$$C_{(T_v)_j}^{2+j} = \frac{(c_v)_j}{\Delta t} + 4\left[2 - (\varepsilon_v)_j \left(1 - \varepsilon_g\right)\right] (\varepsilon_v)_j \sigma \left(T_v^n\right)_j^3 + \rho_{atm} C_p \frac{L_j + S_j}{(r_b)_j}$$

$$+ \rho_{atm} \lambda_{vap} \frac{dq_{sat}^{(T_v)_j}}{d(T_v)_j} \left\{ (f_{wet})_j \frac{L_j + S_j}{(r_b)_j} + \frac{(f_{dry})_j}{L_j} \left(\frac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j + (r_s^{sha})_j} \right) \right\}$$

$$C_{T_g}^{2+j} = -4 (\varepsilon_v)_j \varepsilon_g \sigma \left(T_g^n\right)^3$$

$$F_{(T_v)_j} = (\vec{S}_v^n)_j - (\vec{L}_v^n)_j + \rho_{atm} C_p \frac{L_j + S_j}{(r_b)_j} \left(T_s^n - (T_v^n)_j\right)$$

$$+ \rho_{atm} \lambda_{vap} \left[\begin{array}{l} (f_{wet})_j \dfrac{L_j + S_j}{(r_b)_j} \\ + \dfrac{(f_{dry})_j}{L_j} \left(\dfrac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \dfrac{L_j^{sha}}{(r_b)_j + (r_s^{sha})_j} \right) \end{array} \right] \left(q_s^n - q_{sat}^{(T_v^n)_j}\right) \qquad \text{(Eq. 3.7)}$$

$$C_{T_s}^{3+npft} = -\sum_{j=1}^{npft} \left[(wt)_j \frac{\rho_{atm} C_p}{(r_{ah'})_j} \right]$$

$$C_{q_s}^{3+npft} = -\sum_{j=1}^{npft} \left[(wt)_j \frac{\rho_{atm} \lambda_{vap/sub}}{(r_{aw'})_j} \right]$$

$$C_{(T_v)_j}^{3+npft} \Big|_{j=1}^{npft} = -(wt)_j 4 \varepsilon_v \varepsilon_g \sigma \left(T_v^n\right)_j^3 \quad \text{for} \quad j = 1,2,\ldots,npft$$

$$C_{T_g}^{3+npft} = \sum_{j=1}^{npft} \left[(wt)_j \left\{ \begin{array}{l} \dfrac{c_{soil,1+snl}}{\Delta t} + \dfrac{\rho_{atm} C_p}{(r_{ah'})_j} + \dfrac{\rho_{atm} \lambda_{vap/sub}}{(r_{aw'})_j} \dfrac{dq_g}{dT_g} \\ + 4 \varepsilon_g \sigma \left(T_g^n\right)^3 - \dfrac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl} - z_{1+snl}} \end{array} \right\} \right]$$

$$F_{T_g} = \sum_{j=1}^{npft} \left[(wt)_j \left(\begin{array}{l} \vec{S}_g^n - \vec{L}_g^n + \dfrac{\rho_{atm} C_p}{(r_{ah'})_j} \left(T_s^n - T_g^n\right) + \dfrac{\rho_{atm} \lambda_{vap/sub}}{(r_{aw'})_j} \left(q_s^n - q_g^n\right) \\ + \dfrac{\lambda\left[z_{h,1+snl}\right]}{z_{2+snl} - z_{1+snl}} \left(T_g^n - T_{2+snl}^n\right) \end{array} \right) \right] \qquad \text{(Eq. 4.7)}$$

Δz (in Eqs. 1.7 and 2.7) is a new variable in CLM4 defined as the height at the top of the canopy but no less than 1 m.

To avoid evaporating more than the available water, Vidale & Stöckli (2005) limit the per time step change in evaporation relative to the change in humidity or temperature, e.g. $\frac{\partial(\lambda_{vap/sub} E_g)}{\partial T_g}$, according to the actual soil and canopy water. Subsequent tests have shown that it is more effective to limit the evaporation terms rather than their derivatives (R. Stöckli, pers. communication). Furthermore, only canopy evaporation must be limited because transpiration and soil evaporation decline to zero before depleting the soil water.

Here we limit canopy evaporation, $\lambda_{vap} E_v^w$ (W m^{-2}), to 0.5W_{can} (kg m^{-2}) per time step using a new coefficient, β, calculated at time step n:

$$
\begin{aligned}
&0 \le \beta = \frac{(\lambda_{vap} E_v^w)_{\max}}{\lambda_{vap} E_v^w} \le 1 \\
&(\lambda E_v^w)_{\max} = \frac{\alpha \lambda_{vap} W_{can}}{\Delta t}
\end{aligned}
\qquad \text{(Eq. 6)}
$$

where $(\lambda E_v^w)_{\max}$ is the maximum allowed canopy evaporation during this time step and α is a "security constant" equal to 0.5 (Stöckli, pers. communication; Vidale & Stöckli 2005). Using α in the absence of precipitation may lead to infinitesimal amounts of canopy water by not permitting the entire pool to ever evaporate. To avoid this we arbitrarily add canopy water with values less than 1 x 10^{-2} kg m^{-2} to canopy drip, q_{drip}, which brings the water to the ground.

We rewrite Eqs. 2.7 and 3.7 to include, β, which varies by pft j:

$$C_{q_s}^2 = \sum_{j=1}^{npft} \left[(wt)_j \rho_{atm} \left\{ \begin{array}{l} \frac{\lambda_{vap} \Delta z_j}{\Delta t} + \frac{\lambda_{vap}}{(r_{aw})_j} + \frac{\lambda_{vap/sub}}{(r_{aw'})_j} + \lambda_{vap} (f_{wet})_j \frac{L_j + S_j}{(r_b)_j} \beta_j \\ + \frac{\lambda_{vap} (f_{dry})_j}{L_j} \left(\frac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j + (r_s^{sha})_j} \right) \end{array} \right\} \right]$$

$$C_{(T_v)_j}^2 = -(wt)_j \rho_{atm} \lambda_{vap} \frac{dq_{sat}^{(T_v)_j}}{d(T_v)_j} \left\{ \begin{array}{l} (f_{wet})_j \frac{L_j + S_j}{(r_b)_j} \beta_j \\ + \frac{(f_{dry})_j}{L_j} \left(\frac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j + (r_s^{sha})_j} \right) \end{array} \right\}$$

for $j = 1, 2, \ldots, npft$

$$C_{T_g}^2 = -\sum_{j=1}^{npft} \left[(wt)_j \frac{\rho_{atm} \lambda_{vap/sub}}{(r_{aw'})_j} \frac{dq_g}{dT_g} \right]$$

$$F_{q_s} = \sum_{j=1}^{npft} \left[(wt)_j \left(\begin{array}{l} -\rho_{atm} \lambda_{vap} \left\{ \begin{array}{l} (f_{wet})_j \frac{L_j + S_j}{(r_b)_j} \\ + \frac{(f_{dry})_j}{L_j} \left(\frac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j + (r_s^{sha})_j} \right) \end{array} \right\} \left(q_s^n - q_{sat}^{(T_v^n)_j} \right) \\ - \frac{\rho_{atm} \lambda_{vap/sub}}{(r_{aw'})_j} \left(q_s^n - q_g^n \right) + \frac{\rho_{atm} \lambda_{vap}}{(r_{aw})_j} \left(q_{atm}^n - q_s^n \right) \end{array} \right) \right] \quad \text{(Eq. 2.8)}$$

$$C_{T_s}^{2+j} = -\rho_{atm} C_p \frac{L_j + S_j}{(r_b)_j}$$

$$C_{q_s}^{2+j} = -\rho_{atm} \lambda_{vap} \left\{ (f_{wet})_j \frac{L_j + S_j}{(r_b)_j} \beta_j + \frac{(f_{dry})_j}{L_j} \left(\frac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j + (r_s^{sha})_j} \right) \right\}$$

$$C_{(T_v)_j}^{2+j} = \frac{(c_v)_j}{\Delta t} + 4\left[2 - (\varepsilon_v)_j \left(1 - \varepsilon_g\right)\right](\varepsilon_v)_j \sigma \left(T_v^n\right)_j^3 + \rho_{atm} C_p \frac{L_j + S_j}{(r_b)_j}$$

$$+ \rho_{atm} \lambda_{vap} \frac{dq_{sat}^{(T_v)_j}}{d(T_v)_j} \left\{ (f_{wet})_j \frac{L_j + S_j}{(r_b)_j} \beta_j + \frac{(f_{dry})_j}{L_j} \left(\frac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \frac{L_j^{sha}}{(r_b)_j + (r_s^{sha})_j} \right) \right\}$$

$$C_{T_g}^{2+j} = -4(\varepsilon_v)_j \varepsilon_g \sigma \left(T_g^n\right)^3$$

$$F_{(T_v)_j} = (\vec{S}_v^n)_j - (\vec{L}_v^n)_j + \rho_{atm} C_p \frac{L_j + S_j}{(r_b)_j} \left(T_s^n - (T_v^n)_j\right)$$

$$+ \rho_{atm} \lambda_{vap} \begin{bmatrix} (f_{wet})_j \dfrac{L_j + S_j}{(r_b)_j} \\ + \dfrac{(f_{dry})_j}{L_j} \left(\dfrac{L_j^{sun}}{(r_b)_j + (r_s^{sun})_j} + \dfrac{L_j^{sha}}{(r_b)_j + (r_s^{sha})_j} \right) \end{bmatrix} \left(q_s^n - q_{sat}^{(T_v^n)_j}\right) \qquad \text{(Eq. 3.8)}$$

Later in time step n, a tridiagonal matrix solves for the soil and snow temperatures of deeper layers. Subsequently CLM updates these temperatures to account for the effect of water phase changes in the soil and snow. In earlier versions of CLM, this temperature adjustment was applied to T_g, leading to an adjustment of the sensible and latent heat fluxes (Oleson et al. 2004). In CLM4, to conserve energy after the single-step canopy air space solution, we postpone this adjustment by keeping track of two T_g^{n+1} values: (1) one which is consistent with the state of the canopy air space and (2) one which is adjusted for the soil/snow water phase changes. The latter enters the canopy air space matrix of the following time step as T_g^n. Note that the tridiagonal matrix, which used to solve for all soil/snow temperatures in earlier versions of CLM, now solves for all layers but the top (Figure 2 vs. Figure 1).

Using the LAPACK matrix solver DGESV, CLM solves the set of Eqs. 1 to 4 once per time step *n* for each column in a land grid cell. A grid cell's lake, wetland, glacier, urban, and soil fraction each occupies a separate land unit, each with one column in CLM4. The wetland and glacier land units are treated as bare ground for the purposes of this matrix. For the lake and urban land units CLM uses the existing iterative method instead of the new matrix.

We rewrite Eq. 5 for the general case of multiple pfts, *j*:

$$\begin{pmatrix} C_{T_s}^{1} & 0 & C_{(T_v)_j}^{1} & C_{T_g}^{1} \\ 0 & C_{q_s}^{2} & C_{(T_v)_j}^{2} & C_{T_g}^{2} \\ C_{T_s}^{2+j} & C_{q_s}^{2+j} & C_{(T_v)_j}^{2+j} & C_{T_g}^{2+j} \\ C_{T_s}^{3+npft} & C_{q_s}^{3+npft} & C_{(T_v)_j}^{3+npft} & C_{T_g}^{3+npft} \end{pmatrix} \times \begin{bmatrix} \Delta T_s \\ \Delta q_s \\ \Delta (T_v)_j \\ \Delta T_g \end{bmatrix} = \begin{bmatrix} F_{T_s} \\ F_{q_s} \\ F_{(T_v)_j} \\ F_{T_g} \end{bmatrix} \qquad \text{(Eq. 7)}$$

For bare ground, the rows and columns corresponding to T_v drop out of the matrix. CLM adjusts the size of matrix *A* in every soil column according to the actual number of pfts. The matrix size can range from 3x3 for a soil column with no pfts (i.e., wetland, glacier, or bare soil, where *npft* = 1 but $L + S = 0$) up to 7x7 for a soil column with four pfts.

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Figure 1. CLM3 radiation and heat flux diagram where the resistances denote sensible, latent, or soil/snow heat flux pathways: (A) Sensible and (B) latent heat fluxes for the CLM3 diagnostic canopy air space of a soil column that, for illustration, includes two plant functional types (pfts with index *j*) and bare ground. (A) and (B) also show the vegetation's net solar and longwave radiation terms to illustrate the complete system of Eqs. B, F, and H solving for T_s, T_v, and q_s. (C) Soil/snow heat fluxes for the same soil column, where *i* is the layer index. For the top soil/snow layer, *i* = 1 + *snl*, where *snl* is the number of snow layers from 0 to -5. For the bottom soil layer, *i* = *N*, which in CLM equals 10. Diagram (C) represents CLM's separate matrix of equations solving for T_i. The heat flux into the top layer is given by $G = \vec{S}_g - \vec{L}_g - H_g - \lambda_{vap/sub} E_g$ (Oleson et al. 2004), where $\vec{S}_g$ and $\vec{L}_g$ are the ground's net solar and net longwave radiation fluxes.

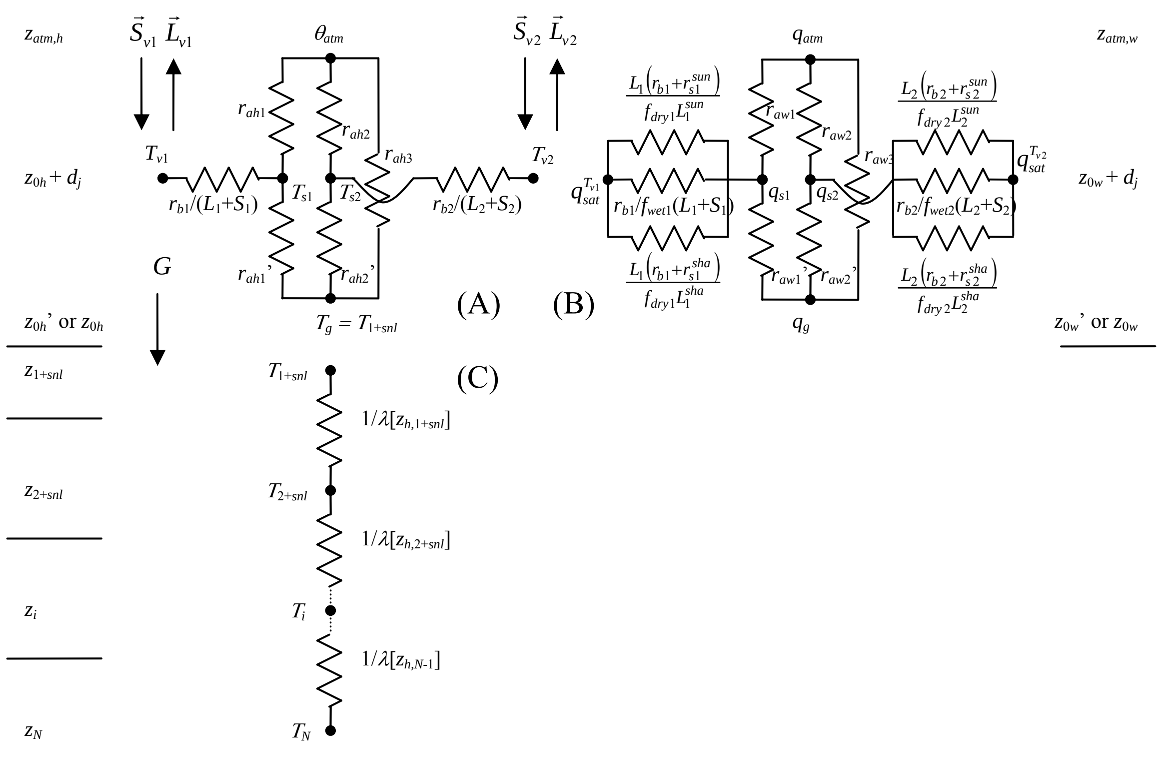

Figure 2. As Figure 1 but for the CLM4 prognostic canopy air space. Diagrams (A) and (B) illustrate the complete system of Eqs. 1 to 4 solving for T_s, T_v, q_s, and T_g. Note that T_s and q_s now are column level variables. The canopy air space and the vegetation now have non-negligible heat and water vapor capacities which introduce storage terms (not shown). Also the same system of equations that solves for the canopy air space now solves for T_g. (C) Now the soil temperature equations solve for T_i from i = 2 + snl to i = N.