APPENDIX F
MULTIPLE-DOPPLER RADAR WIND SYNTHESIS IN CEDRIC

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1. Cartesian components of motion from radial velocities

The normal meteorological coordinate system consists of northward distance y, eastward distance x, and upward distance or height z above mean sea level of a curved earth. The velocity components corresponding to these coordinates are v, u, and w. All distances are in kilometers, and all velocities are in meters per second. The mapping to height above mean sea level is made in the interpolation (SPRINT) and analysis (CEDRIC) software packages, and it will not be explicitly listed here. Usually Doppler radars detect particles so there is an additional component of motion, fallspeed of particles in still air, to be considered. The normal convention of positive radial velocity away from the radar is used.

This mathematical formulation comes from the lead author's unpublished notes that have been accumulated over the years, and parallels the formulations presented by Armijo (1969), Bohne and Srivastava (1975), Ray et al. (1978) and Ray et al. (1980).

a. Mathematical formulation

The projection of particle motion $(\hat{u}, \hat{v}, \hat{w}, \hat{w}_t)$ along the Doppler radar radial direction is

$$\hat{v}_r = \hat{u}\ sin\ a\ cos\ e\ +\ \hat{v}\ cos\ a\ cos\ e\ +\ (\widehat{w + w_t})\ sin\ e, \tag{1}$$

where a and e are the azimuth and elevation angles of the beam. The azimuth angle is measured clockwise from true north (the positive y axis), and the elevation angle is measured upward from the horizontal plane through the radar. The circumflex indicates an average value within the radar sample volume defined by pulse length and beamwidth. Two or more radar measurements are usually combined along with the mass continuity equation to obtain the unknown components of air motion $(\hat{u}, \hat{v}, \hat{w})$. In the case of only two radar measurements additional information about the precipitation fallspeed, $\hat{w}_t$, is needed to obtain a solution for the four unknown components of motion. A similar procedure is followed with three or more radar measurements to separate the vertical air motion from the precipitation fallspeed.

It is more convenient to express the radial velocity in terms of Cartesian coordinates since solutions are obtained after interpolation of the radar measurements at locations in spherical coordinates (r, a, e). Further, if the radar echo (storm) is moving with components (U, V), and the radial velocity samples are taken at time $t + \Delta t$, the above equation is replaced with (adapted from Gal-Chen, 1982)

$$\frac{[\hat{v}_r r]_{t+\Delta t}}{[r]_t} = \left[\hat{u}\left(\frac{x - x_o + U\Delta t}{r}\right) + \hat{v}\left(\frac{y - y_o + V\Delta t}{r}\right) + \widehat{W}\left(\frac{z - z_o}{r}\right)\right]_t, \tag{2}$$

for a radar at (x_o, y_o, z_o) with slant range

$$r = \left[(x - x_o)^2 + (y - y_o)^2 + (z - z_o)^2\right]^{1/2}.$$

Radial velocities are first multiplied by slant ranges from the radar at the sample time $t + \Delta t$. This field is then advected at the storm motion to new locations, where it is divided by slant ranges at the synthesis time t. The coefficients of $\hat{u}$ and $\hat{v}$ on the right hand side of (2) have been modified to account for a change in radar pointing direction.

For M radars, the linear system of equations to be solved at time t is

$$\hat{u}a_m + \hat{v}b_m + \widehat{W}c_m = d_m \ ; \ m = 1,\ 2,\ 3,\ \ldots, \tag{3}$$

where (a_m, b_m, c_m) are the coefficients and d_m is the term involving radial velocity on left hand side in (2), a set for each radar. In the method of least-squares, the error equation

$$Q = \Sigma E_m^2 = \Sigma(\hat{u}a_m + \hat{v}b_m + \widehat{W}c_m - d_m)^2 \tag{4}$$

is minimized with respect to the unknown quantities $(\hat{u},\ \hat{v}\ , \widehat{W})$. The resulting normal equations to be solved for three or two unknown quantities are:

$$\begin{aligned} \hat{u}\Sigma a_m a_m + \hat{v}\Sigma a_m b_m + \widehat{W}\Sigma a_m c_m &= \Sigma a_m d_m \\ \hat{u}\Sigma b_m a_m + \hat{v}\Sigma b_m b_m + \widehat{W}\Sigma b_m c_m &= \Sigma b_m d_m \\ \hat{u}\Sigma c_m a_m + \hat{v}\Sigma c_m b_m + \widehat{W}\Sigma c_m c_m &= \Sigma c_m d_m, \end{aligned} \tag{5}$$

or

$$\begin{aligned} \hat{u}\Sigma a_m a_m + \hat{v}\Sigma a_m b_m &= \Sigma a_m d_m - \widehat{W}\Sigma a_m c_m \\ \hat{u}\Sigma a_m b_m + \hat{v}\Sigma b_m b_m &= \Sigma b_m d_m - \widehat{W}\Sigma b_m c_m. \end{aligned} \tag{6}$$

For the linear system of three equations

$$\begin{aligned} \hat{u}\ A_1 + \hat{v}\ B_1 + \widehat{W}C_1 &= D_1 \\ \hat{u}\ A_2 + \hat{v}\ B_2 + \widehat{W}C_2 &= D_2 \\ \hat{u}\ A_3 + \hat{v}\ B_3 + \widehat{W}C_3 &= D_3, \end{aligned} \tag{7}$$

the solutions are:

$$\begin{aligned} \hat{u} &= D^{-1}\big[D_1(B_2C_3 - B_3C_2) - D_2(B_1C_3 - B_3C_1) + D_3(B_1C_2 - B_2C_1)\big] \\ \hat{v} &= D^{-1}\big[D_1(A_3C_2 - A_2C_3) - D_2(A_3C_1 - A_1C_3) + D_3(A_2C_1 - A_1C_2)\big] \\ \hat{W} &= D^{-1}\big[D_1(A_2B_3 - A_3B_2) - D_2(A_1B_3 - A_3B_1) + D_3(A_1B_2 - A_2B_1)\big], \end{aligned} \tag{8}$$

where the determinant of coefficients

$$D = A_1(B_2C_3 - B_3C_2) - A_2(B_1C_3 - B_3C_1) + A_3(B_1C_2 - B_2C_1).$$

The vertical component $\widehat{W}$ can be separated into air motion and fallspeed either by using $(\hat{u}, \hat{v})$ in the mass continuity equation to obtain $\hat{w}$ or by calculating fallspeed from radar reflectivity factor (or some other means).

For the system of two equations

$$\begin{aligned} \hat{u}\ A_1 + \hat{v}\ B_1 &= D_1 - \widehat{W}C_1 \\ \hat{u}\ A_2 + \hat{v}\ B_2 &= D_2 - \widehat{W}C_2, \end{aligned} \tag{9}$$

the solutions are

$$\begin{aligned} \hat{u} &= \frac{D_1B_2 - D_2B_1}{D} + \widehat{W}\frac{B_1C_2 - B_2C_1}{D} = u' + \epsilon_u\widehat{W} \\ \hat{v} &= \frac{D_2A_1 - D_1A_2}{D} + \widehat{W}\frac{A_2C_1 - A_1C_2}{D} = v' + \epsilon_v\widehat{W}, \end{aligned} \tag{10}$$

where the determinant of coefficients

$$D = A_1B_2 - A_2B_1.$$

The quantities $\hat{u}$ and $\hat{v}$ depend not only on the radar measurements through (u', v') but also on an unknown $\widehat{W}$. The so-called over-determined, dual-Doppler approximations (e.g., Kessinger et al. 1987) use (u', v') as estimates of $(\hat{u}, \hat{v})$. Since ϵ_u and ϵ_v depend only on geometry, the impact of neglecting the vertical component in (10) can be assessed if the bounds of $\widehat{W}$ can be estimated.

It is clear from the preceding equations that the solutions for $(\hat{u}, \hat{v}, \widehat{W})$ are geometrically weighted sums of the interpolated radial velocities. Since the different radar radial velocity measurement errors are independent (the error in a particular radar measurement does not depend on other radar measurement errors even though the measurements themselves are related), the variance of the solutions given by (8) and (10) can be written as sums of radial velocity variance, weighted by the square of the geometric terms. From (8),

$$\begin{aligned} \sigma^2(\hat{u}) &= \Sigma g_{um}^2 \sigma^2(\hat{v}_m) \\ \sigma^2(\hat{v}) &= \Sigma g_{vm}^2 \sigma^2(\hat{v}_m) \\ \sigma^2(\widehat{W}) &= \Sigma g_{Wm}^2 \sigma^2(\hat{v}_m), \end{aligned} \tag{11}$$

which, if all radial velocity variances $\sigma^2(\hat{v}_r)$ are equal, can be rewritten as normalized variances

$$\begin{aligned} \sigma_N^2(\hat{u}) &= \frac{\sigma^2(\hat{u})}{\sigma^2(\hat{v}_r)} = \Sigma g_{um}^2 \\ \sigma_N^2(\hat{v}) &= \frac{\sigma^2(\hat{v})}{\sigma^2(\hat{v}_r)} = \Sigma g_{vm}^2 \\ \sigma_N^2(\widehat{W}) &= \frac{\sigma^2(\widehat{W})}{\sigma^2(\hat{v}_r)} = \Sigma g_{Wm}^2. \end{aligned} \tag{12}$$

Likewise, from (10)

$$\begin{aligned} \sigma_N^2(u') &= \frac{\sigma^2(u')}{\sigma^2(\hat{v}_r)} = \Sigma g_{um}^2 \\ \sigma_N^2(v') &= \frac{\sigma^2(v')}{\sigma^2(\hat{v}_r)} = \Sigma g_{vm}^2. \end{aligned} \tag{13}$$

These normalized variances can be used to estimate the impact of geometry on the transformation from non-orthogonal radial velocities to the orthogonal Cartesian components. This geometric transformation is such that errors in the solutions generally exceed errors in the measured radial velocities.

Once horizontal wind components are found, the mass continuity equation can be used to obtain the vertical component of air motion. There are two ways to proceed, the first uses

$$\frac{\partial(\rho w)}{\partial z} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0,$$

so that

$$\int_{z_k}^{z_{k+1}} \frac{\partial(\rho w)}{\partial z} dz = -\int_{z_k}^{z_{k+1}} \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) dz$$

or, in finite difference form,

$$(\rho w)_c = (\rho w)_p - \delta \Delta z \overline{\left[\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\right]}_{p-c}, \tag{14}$$

where

$$\delta = \begin{cases} +1, & \text{for upward integration} \\ -1, & \text{for downward integration.} \end{cases}$$

The overbar represents an average of divergence values from the previous (p) and current (c) levels. The boundary condition $(\rho w)_b$ must be specified at every (x, y, z_b) at the bottom of the domain if integrating upward or at the top of the domain if integrating downward. This can be done by setting the boundary value to a constant, by using a fraction of the first level divergence times a height increment, or by constructing a field using options such as found in the FUNCTION command. Equation (14) can also be integrated using a variational scheme where both upper and lower boundary conditions are specified within each integration column. Vertical motions rather than density-weighted values are output, where the user specifies the density weighting (an exponential form is provided, but another form can be constructed with the FUNCTION command).

An alternative path is available that is more mathematically rigorous when (u, v) come from the two-equation solution. The first step is to correct (u', v') for precipitation fallspeed using (10),

$$u_2 = u' + \epsilon_u \hat{w}_t$$
$$v_2 = v' + \epsilon_v \hat{w}_t,$$

then substitute

$$u = u_2 + \epsilon_u w$$
$$v = v_2 + \epsilon_v w$$

into the mass continuity equation and integrate. The proper form of the mass continuity equation becomes

$$\frac{\partial(\rho w)}{\partial z} + \frac{\partial(\rho u_2)}{\partial x} + \frac{\partial(\rho v_2)}{\partial y} + \frac{\partial(\rho \epsilon_u w)}{\partial x} + \frac{\partial(\rho \epsilon_v w)}{\partial y} = 0.$$

This partial differential equation can no longer be solved as done in (14) since the desired vertical component appears in the two terms involving horizontal derivatives. These terms cause the solutions for w at adjacent (x, y) grid points to be coupled. In finite difference form, the mass continuity equation becomes

$$\underbrace{(\rho w)_c}_{A} = \underbrace{(\rho w)_p}_{B} \underbrace{- \delta\Delta z \overline{\left[\rho\left(\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y}\right)\right]}_{p-c}}_{C} \underbrace{- \delta\Delta z \overline{\left[\frac{\partial(\rho \epsilon_u w)}{\partial x} + \frac{\partial(\rho \epsilon_v w)}{\partial y}\right]}_{p-c}}_{D}. \tag{15a}$$

Term D consists of two parts:

$$\underbrace{-\frac{\delta\Delta z}{2}\left[\frac{\partial(\rho \epsilon_u w)}{\partial x} + \frac{\partial(\rho \epsilon_v w)}{\partial y}\right]_p}_{D1} \underbrace{- \frac{\delta\Delta z}{2}\left[\frac{\partial(\rho \epsilon_u w)}{\partial x} + \frac{\partial(\rho \epsilon_v w)}{\partial y}\right]_c}_{D2}, \tag{15b}$$

where $D1$ is a function of horizontal derivatives at the previous level and $D2$ is a function of horizontal derivatives at the current level of integration.

To understand this integration scheme, start at a level where boundary conditions have first been applied. The horizontal divergence in (15a) is adjusted in the manner dictated by term $D1$, and $(\rho w)_c$ is estimated by integrating one level in the vertical direction. These first estimates of vertical motion at the current level are then multiplied by the (ϵ_u, ϵ_v) geometric factors and differentiated according to $D2$ in (15b) to readjust the layer convergence, and (15a) is reintegrated. Since only A and $D2$ change, for the nth iteration

$$A_n = B + C + D1 + D2_{n-1},$$

which proceeds from the previous to current levels until A in (15a) stops changing. The change in A is calculated at each grid point, and these differences are globally averaged for comparison with ϵ_o, a parameter set by the user:

$$\mid \overline{(\rho w)_{c,n} - (\rho w)_{c,n-1}} \mid \leq \epsilon_o.$$

Once this condition is satisfied, iterative integration is done from the current to the next level and so on until all levels have been integrated.

b. Synthesis tests

The synthesis in CEDRIC outputs several fields, depending on whether the three-equation or the two-equation solution is selected:

U: component of motion in the x direction, either $\hat{u}$ in (8) or u' in (10),

V: component of motion in the y direction, either $\hat{v}$ in (8) or v' in (10),

W: upward component of motion, $\widehat{W}$ in (8),

USTD: normalized standard deviation, square root of normalized u-variance in (12) or (13),

VSTD: normalized standard deviation, square root of normalized v-variance in (12) or (13),

WSTD: normalized standard deviation, square root of normalized W-variance in (12), available only when three-equation solution is selected,

EWU: geometric factor that multiplies vertical component $\widehat{W}$ in (10), available only when two-equation solution is selected,

EWV: geometric factor that multiplies vertical component $\widehat{W}$ in (10), available only when two-equation solution is selected.

The user specifies input parameters $DTEST1, DTEST2, DTEST3$ against which the above geometric parameters are tested. Selection of the two- or three- equation solution along with these test values determines the synthesis outcome at each (x, y, z) grid point. If only one radial velocity is present, no solution exists and all output fields are set to a bad data flag.

Two-equation solution (the number of radars, M ≥2):

if the magnitudes of (EWU, EWV) are both less than $DTEST1$,

and $(USTD, VSTD)$ are both less than $DTEST2$,

the output (U, V) are given by (u', v') in (10).

Three-equation solution is selected (the number of radars, M ≥3):

if $(USTD, VSTD)$ are both less than $DTEST2$,

the output (U, V) are given by $(\hat{u}, \hat{v})$ in (8),

and if $WSTD$ is less than $DTEST3$,

the output W is given by $\widehat{W}$ in (8).

If only two radial velocities are present at the grid point, the procedure follows the one for a two-equation solution.

2. Coplane components of motion from radial velocities

Lhermitte and Miller (1972) first introduced COPLAN radar scanning where measurements are taken in a cylindrical coordinate system that is more natural for two radars (Fig. 1). One axis is along the baseline joining the two radars, another is perpendicular to the baseline, and the third is the coplane (dihedral) angle associated with a series of planes defined by the two radar beams and the horizontal plane passing through the radars. This coordinate system simplifies the mathematical formulation, and it is the same one used by Armijo (1969) in his transformation that was needed to solve the mass continuity equation for vertical motion. Miller and Strauch (1974) expanded on the coplane coordinate concept, including a detailed analysis of the errors involved in the synthesis. Doviak et al. (1976) also dealt with error analysis, and they included errors associated with the integration of the mass continuity equation. A series of papers by French scientists (Testud and Chong; Chong et al.; and Chong and Testud, 1983) revisited all aspects of coplane analysis, including interpolation, synthesis, and integration of the mass continuity equation.

a. Mathematical formulation

When the original radar sampling process is done in coplanes formed by the beams from two radars, a procedure similar to the one for the two-equation solution is followed. If only two radar measurements are available, rewrite (1) as

$$\begin{aligned} \hat{u}\sin a_1' + \hat{v}\cos a_1' &= \frac{\hat{v}_1}{\cos e_1} - \widehat{W}\tan e_1 \\ \hat{u}\sin a_2' + \hat{v}\cos a_2' &= \frac{\hat{v}_2}{\cos e_2} - \widehat{W}\tan e_2, \end{aligned} \tag{16}$$

where the angle $a' = a - a_o$ is measured from the baseline (at azimuth angle a_o) that joins the two radars. Equation (16) has the solution

$$\begin{aligned} \hat{u} &= D^{-1}\left[\frac{\hat{v}_1\cos a_2'}{\cos e_1} - \frac{\hat{v}_2\cos a_1'}{\cos e_2}\right] - \widehat{W}\tan\alpha \\ \hat{v} &= D^{-1}\left[\frac{\hat{v}_2\sin a_1'}{\cos e_2} - \frac{\hat{v}_1\sin a_2'}{\cos e_1}\right], \end{aligned} \tag{17}$$

where $D = \sin(a_1' - a_2') = \sin(a_1 - a_2)$, and the coplane relations (Lhermitte and Miller, 1970)

$$\tan\alpha = \frac{\tan e_i}{\sin a_i'};\ i = 1, 2 \tag{18}$$

have been used. The quantity α is the dihedral angle between the coplane defined by the two intersecting radar beams and the horizontal plane passing through the two radar locations. The radar beam is steered in the elevation angle direction as it is rotated in azimuth for a fixed coplane angle according to (18). Measurements are taken at range-azimuth locations in a series of these planes, with α increasing upwards from the horizontal plane of the two radars.

An alternative to (17) that will uncouple the vertical component from $\hat{u}$ is to introduce coplane coordinates

$$\begin{aligned} x_c &= (x^2 + z^2)^{1/2} \\ y_c &= y \\ \alpha &= Tan^{-1}(z/x), \end{aligned}$$

and solve for components of the motion within coplanes:

$$\begin{aligned} \hat{u}_c(x_c, y_c) &= \frac{r_1\hat{v}_1(y_c - y_{c1}) - r_2\hat{v}_2(y_c - y_{c2})}{2dx_c} \\ \hat{v}_c(x_c, y_c) &= \frac{r_2\hat{v}_2 - r_1\hat{v}_1}{2d}, \end{aligned} \tag{19}$$

where $2d$ is the distance between the two radars located at $(0, y_{c1})$ and $(0, y_{c2})$. The coplane coordinates x_c and y_c are measured perpendicular and parallel to the radar baseline. Since the component of motion perpendicular to the radar beam is not measured by the radar, the component normal to the coplanes does not appear in (19). In coplanes, distances from the radars are

$$\begin{aligned} r_1 &= \left[x_c^2 + (y_c - y_{c1})^2\right]^{1/2} \\ r_2 &= \left[x_c^2 + (y_c + y_{c2})^2\right]^{1/2}. \end{aligned}$$

Normalized variances within each coplane are similar in form to the ones for Cartesian components in (13). The two-equation Cartesian solution (in a relative coordinate system with y along the two-radar baseline) and the coplane solution give identical results in the horizontal plane ($\alpha = 0°$).

After $\hat{u}_c$ is corrected for fallspeed, the coplane components of air motion are:

$$\begin{aligned} u_c' &= \hat{u}_c - \hat{w}_t\sin\alpha \\ v_c' &= \hat{v}_c. \end{aligned}$$

These components are integrated in the mass continuity equation written in the form

$$\frac{1}{x_c}\frac{\partial(\rho w_c')}{\partial\alpha} + \frac{1}{x_c}\frac{\partial}{\partial x_c}(\rho x_c u_c') + \frac{\partial}{\partial y_c}(\rho v_c') = 0 \tag{20}$$

to obtain the component normal to the coplanes. A finite difference form of (20) similar to (14) is used, with an exponential density weighting provided. Once these coplane components are calculated, horizontal wind components are obtained:

$$\begin{aligned}\hat{u} &= u_c' \cos\alpha - w_c' \sin\alpha \\ \hat{v} &= v_c' \\ \hat{w} &= u_c' \sin\alpha + w_c' \cos\alpha.\end{aligned} \tag{21}$$

The velocity transformation in (21) is done when the winds at coplane coordinates are interpolated to Cartesian coordinates with the REMAP command.

b. Synthesis tests

For coplane synthesis, CEDRIC outputs a subset of fields:

U: component of motion perpendicular to the baseline, $\hat{u}_c$ in (19)

V: component of motion parallel to the baseline, $\hat{v}_c$ in (19)

USTD: normalized standard deviation, similar to the square root of normalized u-variance in (13),

VSTD: normalized standard deviation, similar to the square root of normalized v-variance in (13),

The user specifies input parameters $DTEST2$ and $DTEST3$ against which the above geometric parameters are tested. Selection of coplane coordinate option (housekeeping words are tested or the COORD command is used) along with these test values determines the synthesis outcome at each (x_c, y_c, c) grid point. If only one radial velocity is present, no solution exists and all output fields are set to a bad data flag.

Two-radar coplane solution (the number of radars, M = 2):

if $(USTD, VSTD)$ are both less than $DTEST2$,

the output (U, V) are given by (u_c, v_c) in (19).

(EWU, EWV) are zero in coplane coordinates so they are not output.

The following steps are taken for multiple-Doppler radar wind synthesis:

1. Interpolate the radar measurements using the SPRINT software package. If measurements were originally taken in the coplane coordinate system, interpolation within these coplanes is selected. If measurements were taken in the normal radar spherical coordinate system, interpolation to Cartesian coordinates should be done. There is little advantage in interpolating to coplanes if the data were not taken in this coordinate system. The SPRINT package has no provision for such interpolation; however, the REORDER package of ATD/RDP does.
2. Select the appropriate coordinate system synthesis, integration and interpolation path in CEDRIC. This is done automatically if the appropriate housekeeping words are correctly set. Otherwise, the user must invoke the COORD command to force the correct choice of mathematical formulation.
3. Specify the necessary parameters in the SYNTHES command in CEDRIC. The three-equation solution fields come from (8) and (12). The Cartesian synthesis allows for an over-determined, two-equation solution with the various fields given by (10) and (13). For coplane synthesis at coplane coordinates, only two radars are allowed, and the solutions come from (19).
4. Invoke the CONVERGE command to compute horizontal or coplane convergence, the negative of divergence of the horizontal winds in (8) or (10) or the coplane winds in (19).

5. Specify the necessary parameters in the INTEGR command for Cartesian (14) or coplane (20) integration. If the user wishes to use the iterative integration of (15), the MASS2 command is used.

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